# ON THE LINEAR EQUATIONS CALCULATING THE SHALLOW GROUNDWATER TEMPERATURE NEAR THE WATER TABLE FROM THE AIR TEMPERATURE IN THE PADDY FIELDS

Kiyoshi FUKUDA
Katsuhiko IZUTSU and Tadao MAEKAWA

## 1. Introduction

Because the temperature of water and soil directly effects the growth of crops, it is important to know the temperature of water, including shallow groundwater, as well as to know their quantity.

Therefore a lot of investigations have been undertaken by many concerning this problem. Some have trid to construct an equation to calculate the values of soil temperature. For example, AZUMA [5] constructed a linear equation to calculate the yearly average temperature of soil from the yearly average air temperature.

What we have tried here, however, is to construct a linear equation by calculating the values of shallow groundwater temperature (S.G.T.) near the water table from air temperature for a given area for a given month.

The paper presented here shows the results of our efforts. This is the 12th report of "The shallow groundwater in the downstream basin of the Aya River, Kagawa Prefecture".

## 2. Theory

When we plotted the values of the shallow groundwater temperature (S.G.T.) near the water table or $\theta_g$ (°c) ($y$-axis) against the observed values of the air temperature (A.T.) or $\theta_a$ (°c) ($x$-axis) at the area where the values of S.G.T. were observed, we obtained the closed lines showing the $\theta_g - \theta_a$ relationship as shown in Fig.1.

These closed lines told us that we could not use the straight lines (the linear equations) to show the $\theta_g - \theta_a$ relationship if we wanted to construct an equation to show this relationship.

However, if we could find the linear relationship between $\theta_g$ and $\theta_a$ shown in Fig. 1, we could construct a linear equation (simple and most practical) to show the $\theta_g - \theta_a$ relationship.

Therefore, to obtain the linear equations, we divided a year (12 months) (Fig.1) into three periods I, II and III.

They were:

Period I = four months—January to April;
Period II = three months—May to July; and
Period III = five months—August to December.

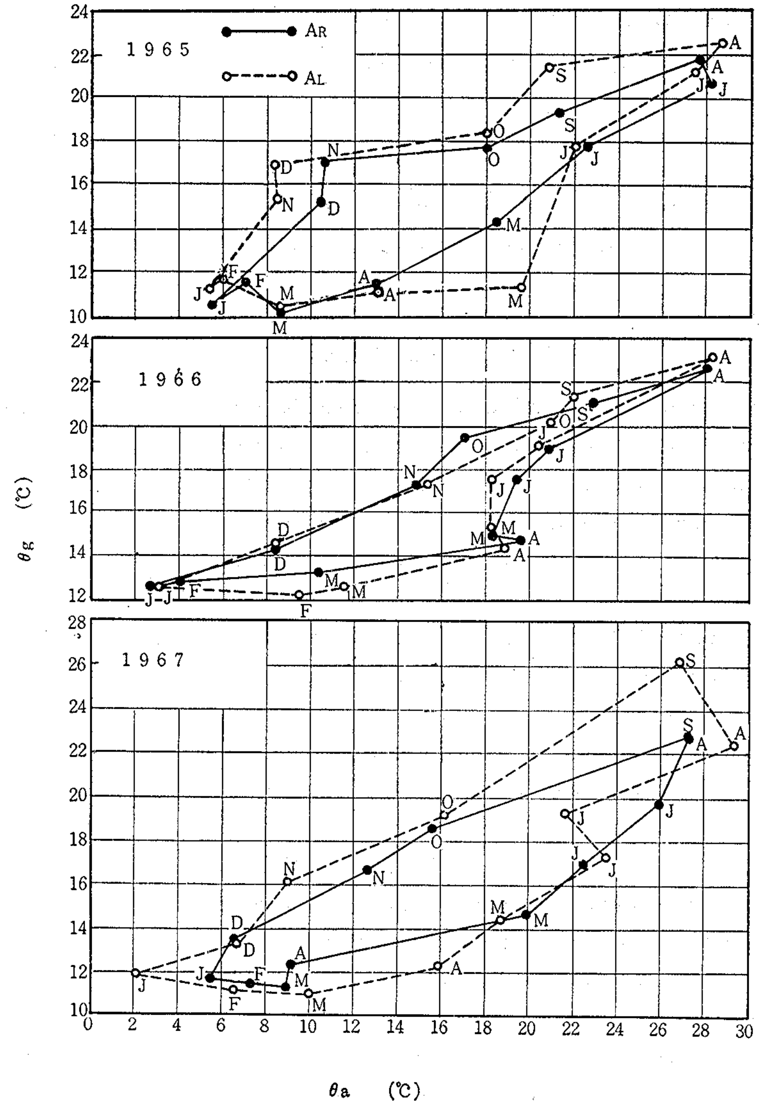


Fig. 1. Variations of observed values of S.G.T. ($\theta$g, °c) for $A_R$ (solid circles) and for $A_L$ (open circles) during 1965 (top figure), 1966 (middle figure) and 1967 (bottom figure). J represents January, F, February and so on.

Through this method, the relationship between $\theta_g$ and $\theta_a$ during each period seemed to be roughly a linear relationship as shown by the straight lines in Figs.3, 4, 5 and 6.

Therefore, for period I, we constructed Eq. (1) to show the $\theta_g - \theta_a$ relationship

$$\theta_{g\,\mathrm{I}} = l\theta_{a\,\mathrm{I}} + b \quad \cdots\cdots(1)$$

For period II, we constructed Eq.(2),

$$\theta_{g\,\mathrm{II}} = m\theta_{a\,\mathrm{II}} + c \quad \cdots\cdots(2)$$

For period III, we also constructed Eq.(3),

$$\theta_{g\,\mathrm{III}} = n\theta_{a\,\mathrm{III}} + d, \quad \cdots\cdots(3)$$

where

$\theta_{g\,\mathrm{I}}$, $\theta_{g\,\mathrm{II}}$ and $\theta_{g\,\mathrm{III}}$ = The values of S.G.T. or $\theta_g$ for the periods I, II and III respectively, (°c).

$\theta_{a\,\mathrm{I}}$, $\theta_{a\,\mathrm{II}}$ and $\theta_{a\,\mathrm{III}}$ = The air temperature for the periods I, II and III respectively, (°c).

*l, m* and *n* = The coefficients of $\theta_a$ for the periods I, II and III respectively.

*b, c* and *d* = The constants for the periods I, II and III respectively.

To calculate the values of *l, m* and *n,* and *b, c* and *d,* we used the method of least squares [8] and constructed Eqs.(4), (5) and (6).

$$l=\frac{\overline{\theta_{g\,\mathrm{I}}\,\theta_{a\,\mathrm{I}}}-\overline{\theta_{g\,\mathrm{I}}}\;\overline{\theta_{a\,\mathrm{I}}}}{\overline{(\theta_{a\,\mathrm{I}}{}^{2})}-(\overline{\theta_{a\,\mathrm{I}}})^{2}},\quad b=\overline{\theta_{g\,\mathrm{I}}}-l\overline{\theta_{a\,\mathrm{I}}}\quad\cdots\cdots(4)$$

$$m=\frac{\overline{\theta_{g\,\mathrm{II}}\,\theta_{a\,\mathrm{II}}}-\overline{\theta_{g\,\mathrm{II}}}\;\overline{\theta_{a\,\mathrm{II}}}}{\overline{(\theta_{a\,\mathrm{II}}{}^{2})}-(\overline{\theta_{a\,\mathrm{II}}})^{2}},\quad c=\overline{\theta_{g\,\mathrm{II}}}-m\overline{\theta_{a\,\mathrm{II}}}\quad\cdots\cdots(5)$$

$$n=\frac{\overline{\theta_{g\,\mathrm{III}}\,\theta_{a\,\mathrm{III}}}-\overline{\theta_{g\,\mathrm{III}}}\;\overline{\theta_{a\,\mathrm{III}}}}{\overline{(\theta_{a\,\mathrm{III}}{}^{2})}-(\overline{\theta_{a\,\mathrm{III}}})^{2}},\quad d=\overline{\theta_{g\,\mathrm{III}}}-n\overline{\theta_{a\,\mathrm{III}}}\quad\cdots\cdots(6)$$

### 3. Data

The values of $\theta_g$ were the arithmetic mean of the field data obtained from 51 observation wells (crosses in Fig.2). We referred to these values as the average values of $\theta_g$ of the regions AR and AL on the day that observations were made. The field data was obtained from the shallow groundwater investigations that we have been carrying out ever since July 1964 by using 51 wells in the downstream basin of the Aya River, Kagawa Prefecture [1, 2, 3, 4, 6] (Fig.2).

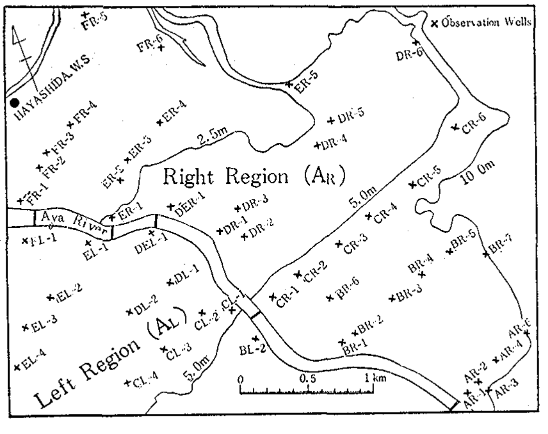


Fig. 2. Simplified map of the study area showing the observation wells (crosses) where the S.G.T. were measured, and also showing the HAYASHIDA (WEATHER STATION) (solid circle). The data on air temperature that was used in this paper was observed by this station.

The values of $\theta_a$ were calculated by using Eq.(7),

$$\theta_a = \tfrac{1}{2}\,(\theta_{ah} + \theta_{al})\quad\cdots\cdots(7)$$

where

$\theta_{ah}$, $\theta_{al}$ = The highest and the lowest air temperature on the day that the field observations of S.G.T. were made respectively, (°c).

We referred to the values of $\theta_a$ as the mean-daily air temperature on the day that the observations of S.G.T. were made.

For the data showing $\theta_{ah}$ and $\theta_{al}$, we used the data reported by the HAYASHIDA (WEATHER STATION) (7) (solid circle in Fig.2).

The values of $\theta_g$ and $\theta_a$ were shown by the solid circles for AR and the open circles for AL for 1965 (top figure), 1966 (middle figure) and 1967 (bottom figure) in Fig.1.

## 4. Results and Discussion

Using Eqs.(4), (5) and (6), we calculated the values of the coefficients (*l, m* and *n*) and the constants (*b, c* and *d*), and the results were listed in Table 1.

Table 1 Values of *l,m,n,b,c* and *d*.

| | | *l* | *m* | *n* | *b* | *c* | *d* |
|---|---|---|---|---|---|---|---|
| 1965 | AR | 0.08682 | 0.60249 | 0.300 | 10.1851 | 3.7492 | 12.942 |
| | AL | −0.01174 | 1.1529 | 0.333 | 11.3721 | −9.7127 | 13.259 |
| 1966 | AR | 0.1803 | 1.6208 | 0.267 | 11.6503 | −14.5726 | 14.025 |
| | AL | 0.1231 | 0.3169 | 0.451 | 11.6078 | 11.2578 | 10.713 |
| 1967 | AR | 0.0624 | 0.8329 | 0.440 | 11.0945 | −1.8624 | 11.0153 |
| | AL | 0.06190 | 0.6941 | 0.471 | 10.9988 | 2.3082 | 11.196 |
| 1965— | AR | 0.1177 | 0.8378 | 0.407 | 10.9393 | −0.9890 | 11.374 |
| 1967* | AL | 0.06631 | 1.2336 | 0.423 | 11.3029 | −8.6737 | 11.652 |

*The average of the three years from 1965 to 1967.

From the values listed in Table 1, we were able to build the equations for calculating values of $\theta_g$ for a given region for a given period, and then for a given month.

For the right region (AR) for periods I, II and III in 1965, with the values of $l = 0.08682$, $b = 10.1851$; $m = 0.60249$, $c = 3.7492$; and $n = 0.300$, $d = 12.942$ (Table 1), we constructed Eqs. (8), (9) and (10) respectively.

$$\theta_{g\,I}]AR = 0.08682\,[\theta_{a\,I}]\,AR + 10.1851 \quad \cdots\cdots(8)$$
$$\theta_{g\,II}]AR = 0.60249\,[\theta_{a\,II}]\,AR + 3.7492 \quad \cdots\cdots(9)$$
$$\theta_{g\,III}]AR = 0.300\,[\theta_{a\,III}]\,AR + 12.942 \quad \cdots\cdots(10)$$

For the left region (AL), with the values of $l = -0.01174$, $b = 11.3721$; $m = 1.1529$, $c = -9.7127$; and $n = 0.333$, $d = 13.259$, we constructed Eqs.(11), (12) and (13).

$$\theta_{g\,I}]AL = -0.01174\,[\theta_{a\,I}]\,AL + 11.3721 \quad \cdots\cdots(11)$$
$$\theta_{g\,II}]AL = 1.1529\,[\theta_{a\,II}]\,AL - 9.7127 \quad \cdots\cdots(12)$$
$$\theta_{g\,III}]AL = 0.333\,[\theta_{a\,III}]\,AL + 13.259 \quad \cdots\cdots(13)$$

In like manner, we constructed Eqs.(14), (15) and (16) for AR; and Eqs.(17), (18) and (19) for AL during 1966.

$$\theta_{g\,I}]AR = 0.1803\,[\theta_{a\,I}]AR + 11.6503 \quad \cdots\cdots(14)$$
$$\theta_{g\,II}]AR = 1.6208\,[\theta_{a\,II}]AR - 14.5726 \quad \cdots\cdots(15)$$
$$\theta_{g\,III}]AR = 0.267\,[\theta_{a\,III}]AR + 14.025 \quad \cdots\cdots(16)$$
$$\theta_{g\,I}]AL = 0.1231\,[\theta_{a\,I}]AL + 11.6078 \quad \cdots\cdots(17)$$
$$\theta_{g\,II}]AL = 0.3169\,[\theta_{a\,II}]AL + 11.2578 \quad \cdots\cdots(18)$$
$$\theta_{g\,III}]AL = 0.451\,[\theta_{a\,III}]AL + 10.713 \quad \cdots\cdots(19)$$

In the same way, we constructed Eqs.(20), (21) and (22)for AR, Eqs.(23), (24) and (25) for AL during 1967.

$$\theta_{g\,I}\,]AR = 0.0624\,[\theta_{a\,I}\,]AR + 11.0945 \quad \cdots\cdots(20)$$
$$\theta_{g\,II}\,]AR = 0.8329\,[\theta_{a\,II}\,]AR - 1.8624 \quad \cdots\cdots(21)$$
$$\theta_{g\,III}\,]AR = 0.440\,[\theta_{a\,III}\,]AR + 11.0153 \quad \cdots\cdots(22)$$
$$\theta_{g\,I}\,]AL = 0.06190\,[\theta_{a\,I}\,]AL + 10.9988 \quad \cdots\cdots(23)$$
$$\theta_{g\,II}\,]AL = 0.6941\,[\theta_{a\,II}\,]AL + 2.3082 \quad \cdots\cdots(24)$$
$$\theta_{g\,III}\,]AL = 0.471\,[\theta_{a\,III}\,]AL + 11.196 \quad \cdots\cdots(25)$$

To show the average of the three years from 1965 to 1967, we constructed Eqs.(26), (27) and (28) for AR; and Eqs.(29), (30) and (31) for AL.

$$\theta_{g\,I}\,]AR = 0.1177\,[\theta_{a\,I}\,]AR + 10.9393 \quad \cdots\cdots(26)$$
$$\theta_{g\,II}\,[AR = 0.8378\,[\theta_{a\,I}\,]AR - 0.9890 \quad \cdots\cdots(27)$$
$$\theta_{g\,III}\,]AR = 0.407\,[\theta_{a\,III}\,]AR + 11.374 \quad \cdots\cdots(28)$$
$$\theta_{g\,I}\,]AL = 0.06631\,[\theta_{a\,I}\,]AL + 11.3029 \quad \cdots\cdots(29)$$
$$\theta_{g\,II}\,]AL = 1.2336\,[\theta_{a\,II}\,]AL - 8.6739 \quad \cdots\cdots(30)$$
$$\theta_{g\,III}\,]AL = 0.423\,[\theta_{a\,III}\,]AL + 11.652 \quad \cdots\cdots(31)$$

Using Eq.(8), we calculated the values of $\theta_g$ for period I and obtained the straight line I as shown in Fig.3(AR). This line showed the calculated values of $\theta_g$ for AR for period I in 1965. The open circles on the line are the calculated values of $\theta_g$ corresponding to the observed ones shown by the solid circles.

Using Eq.(9), we obtained the straight line II as shown in Fig.3 (AR). This line showed the observed values of $\theta_g$ for AR for period II in 1965. Using Eq.(10), we also obtained the straight line III as shown in Fig.3(AR). This line also showed the calculated values of $\theta_g$ for AR for period III in 1965. For AL for periods I, II and III in 1965(using Eqs.(11), (12) and (13) )we drew up the straight lines I, II and III respectively as shown in Fig.3(AL).

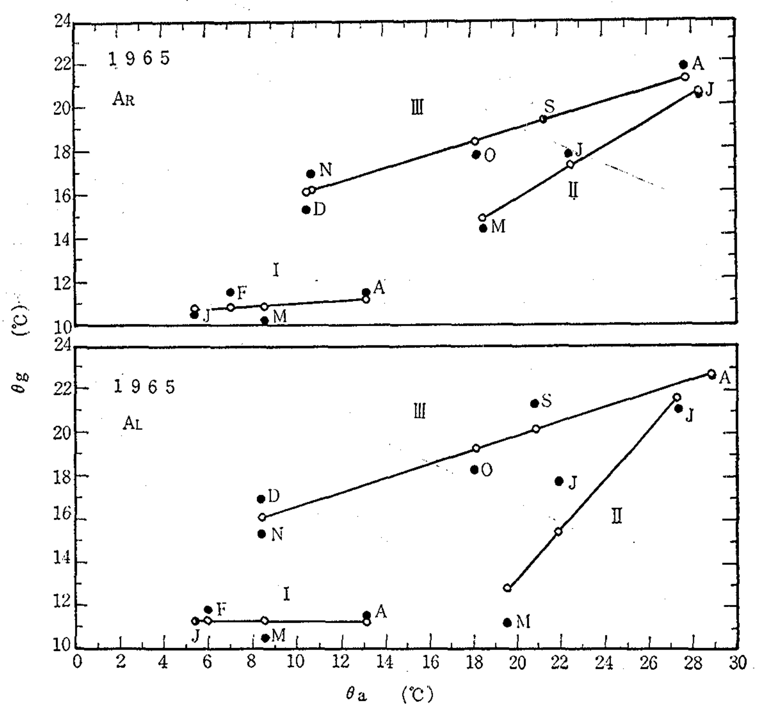


Fig. 3.
Calculated values of S.G.T. (open circles) ($\theta g$, °c) and the observed ones (solid circles) for AR (upper figure) and AL (lower figure) during 1965.
I, II and III mean the periods I, II and III respectively.

In like manner, for the three periods in 1966, we drew up the three straight lines I, II and III (Fig.4(AR)) by using Eqs.(14), (15) and (16) for AR; the three straight lines I, II and III (Fig.4 (AL)) by using Eqs.(17), (18) and (19) for AL.

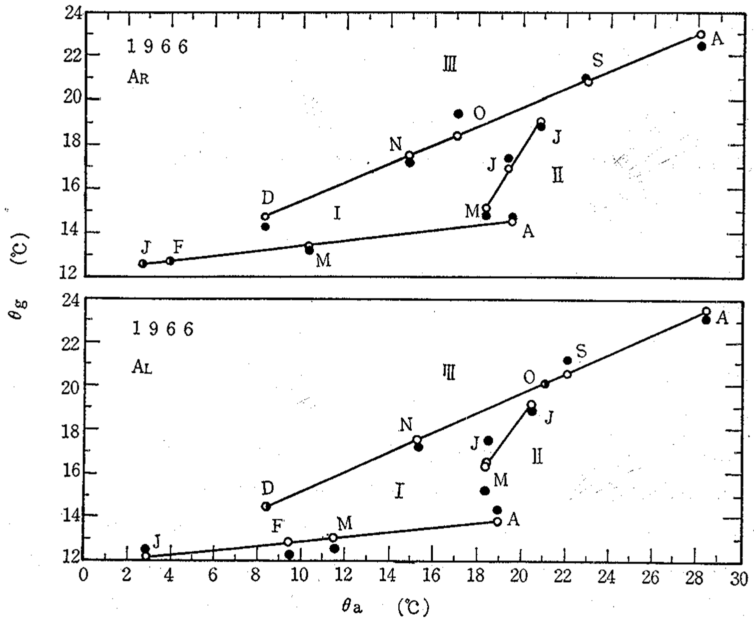


Fig. 4. Calculated values of S.G.T. (θg, °c) (open circles) and the observed ones (solid circles) for AR and AL during 1966. I, II and III have the same meaning as in Fig. 3.

For the year 1967, we also drew up three straight lines I, II and III (Fig.5(AR)) by using Eqs.(20), (21) and (22) for AR; and the three straight lines I, II and III (Fig.5(AL)) by using Eqs.(23), (24) and (25) for AL.

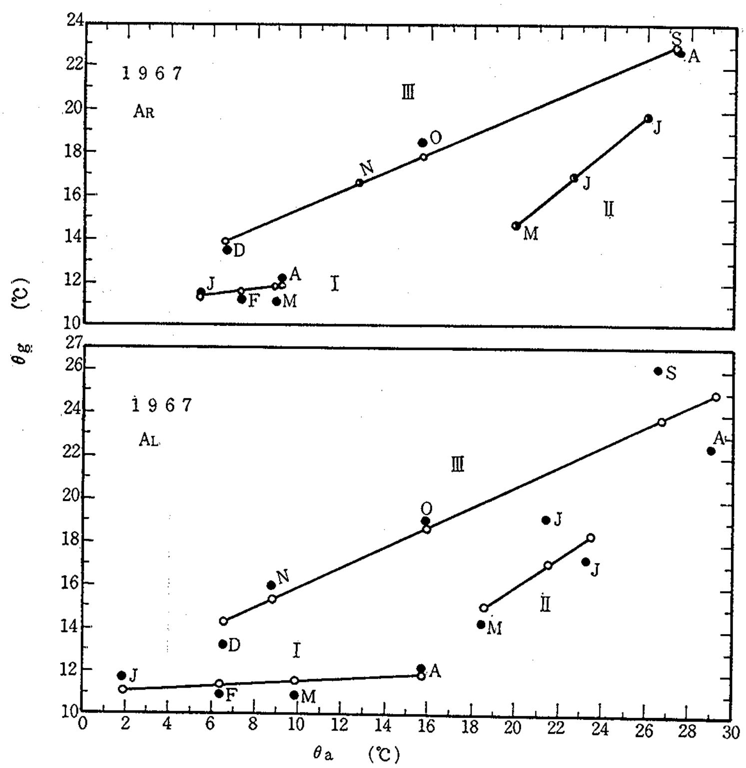


Fig. 5. Calculated values of S.G.T. (θg, °c) (open circles) and the observed ones (solid circles) for AR and AL during 1967. I, II and III have the same meaning as in Fig. 3.

To show the average of the three years, we again drew up three straight lines I, II and III (Fig.6(AR)) by using Eqs.(26), (27) and (28) for AR; and the three straight lines I, II and III (Fig.6(AL)) by using Eqs. (29), (30) and (31) for AL.

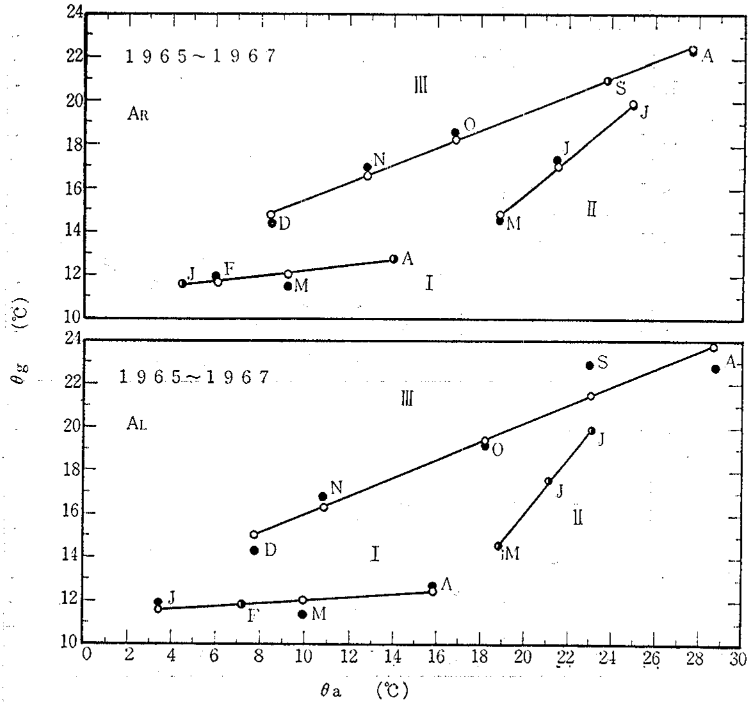


Fig. 6. Calculated values of S.G.T. ($\theta$g, °c) (open circles) and the observed ones (solid circles) for AR and AL for the average of the three years from 1965 to 1967. The meaning of I, II and III is the same as in Fig. 3.

From the graphs shown in Figs.3, 4, 5 and 6, the calculated values of $\theta_g$ (open circles) are seen to be close to the observed values (solid circles).

In order to see how much the calculated values deviated from the observed ones, we used Eq.(32),

$$\varepsilon = \left| \frac{\theta_{gc} - \theta_g}{\theta_g} \right| \times 100(\%) \qquad \text{.......(32)}$$

where $\varepsilon$ = The deviation of the calculated values of $\theta_g$ from the observed ones, (%).

$\theta_{gc}$ = The calculated values of $\theta_g$, (°c).

$\theta_g$ = The observed values of S.G.T., (°c).

The results as calculated by Eq.(32), were shown in Fig.7, and the yearly mean values of them ($\bar{\varepsilon}$) were listed in Table 2.

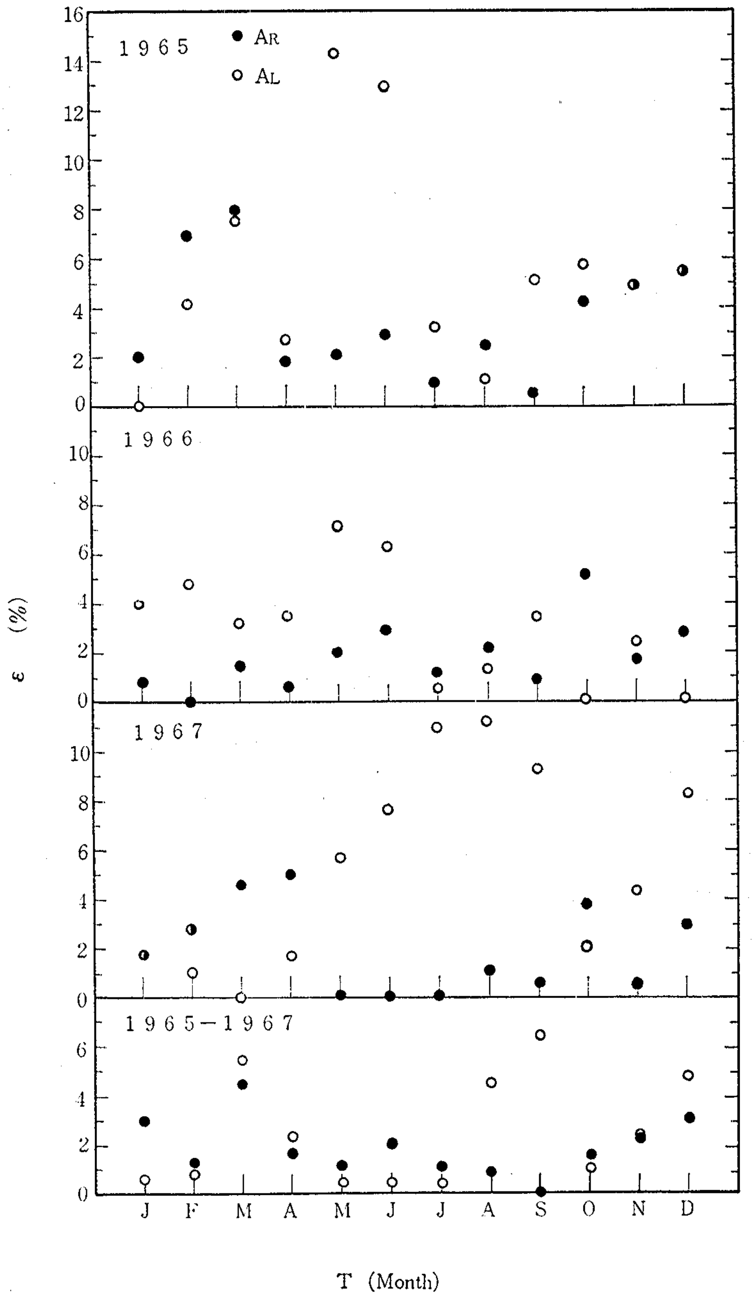


Fig. 7. Variations of the values of $\epsilon$ (%) for AR (solid circles) and AL (open circles) during 1965, 1966, 1967 and the average of the three years.

Table 2 Values of $\bar{\epsilon}$ for AR and AL during 1965, 1966, 1967 and for the average of them.

| | 1965 | | 1966 | | 1967 | | 1965—1967* | |
|---|---|---|---|---|---|---|---|---|
| | AR | AL | AR | AL | AR | AL | AR | AL |
| $\bar{\epsilon}$ (%) | 3.47 | 5.56 | 4.22 | 3.62 | 1.87 | 6.23 | 1.69 | 2.68 |

*The average of the three years from 1965 to 1967.

Except for the extremely high values, 14.2 % (AL, May 1965)~11.0 % (AL, July 1967), most of $\varepsilon$ (%) only deviated a few per cent from the observed values. Furthermore the values of $\varepsilon$ (%) only fell between 1.87% (AR, 1967) and 6.23 % (AL, 1967).

Therefore, Eqs.(8) thru (31) are adequate for calculating the values of S.G.T. from the values of A.T. in the study area.

The relationship between the values of $l$, $m$ and $n$ were

$$l<n<m \qquad (33)$$

through all the graphs of the entire three years and the average of them.

Because the values shown by $l$ were close to zero, it can be said that the values of S.G.T. were roughly parallel to the values of A.T. during the four months from January to April (period I, Figs. 3, 4, 5 and 6).

## 5. Summary

In order to construct a simple and practical equation calculating the shallow groundwater temperature (S.G.T. or $\theta_g$ (°c)) near the water table in the paddy fields from the air temperature (A.T. or $\theta_a$) in the same area, we analysed the data of S.G.T. that was obtained from our field investigations. The results obtained were as follows;

1) We found that the $\theta_g - \theta_a$ relationship could be expressed by three straight lines (or linear equations) when a year (12 months) was divided into three periods - - I, II and III (Figs.3, 4, 5 and 6).

2) To show the three straight lines, I, II and III, we constructed Eqs.(1), (2) and (3) respectively, and calculated the coefficients and constants of Eqs.(1), (2) and (3) by using the method of least squares (Eqs. (4), (5) and (6)). The results were listed in Table 1.

3) From the values listed in Table 1, we constructed Eqs.(8) thru (31) for purpose of calculating the values of $\theta_g$ from $\theta_a$ for AR and AL for every month during 1965, 1966 and 1967; and for the average of the three years from 1965 to 1967.

4) The calculated values of $\theta_g$ (using Eqs.(8) thru (31)) were close to the observed values (Figs.3, 4, 5 and 6). The deviation of the calculated values from the observed ones calculated by using Eq. (32) was only a few per cent in yearly average values.

5) Therefore, Eqs.(8) thru (31) are adequate to use in calculating the values of S.G.T. ($\theta_g$) from the values of A.T. ($\theta_a$) for the study area for the periods. Thus, the original linear equations Eqs.(1), (2) and (3) with Eqs.(4), (5) and (6) can be used to show the relationship between $\theta_g$ and $\theta_a$ for a given area for a given period if the $\theta_g - \theta_a$ relationship becomes the linear relationship.

## 6. Acknowledgements

The writers wish to thank to the Emeritus Professor Dr. Hitoshi FUKUDA and Professor Dr. Hiroyuki OGATA, both of Tokyo University for their encouragement.

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# 水田地帯の地下水面附近の浅層地下水温を気温から計算する1次方程式について

福田　清，井筒勝彦，前川忠夫


## 要　旨

水田地帯の地下水面附近の浅層地下水温を気温から計算するための1次方程式を得た。

気温 $\theta_a$ を $x$ 軸に，地下水温 $\theta_g$ を $y$ 軸に目盛り，各月の値を結ぶと一つの閉曲線を描く (Fig. 1) ので，このままでは $\theta_g$-$\theta_a$ 関係を示す式として直線式は使えない。

そこで1か年を3期（Ⅰ，Ⅱ，およびⅢ）に分け，

Ⅰ は　1～4月　4か月
Ⅱ は　5～7月　3か月
Ⅲ は　8～12月　5か月

とすると，各期では，$\theta_g$-$\theta_a$ 関係は，ほぼ直線関係になることを見出した。

冬期の $\theta_g$-$\theta_a$ 関係を示す方程式として，

$$\theta_{g\,\mathrm{I}} = l\theta_{a\,\mathrm{I}} + b \quad \cdots\cdots(1)$$
$$\theta_{g\,\mathrm{II}} = m\theta_{a\,\mathrm{II}} + c \quad \cdots\cdots(2)$$
$$\theta_{g\,\mathrm{III}} = n\theta_{a\,\mathrm{III}} + d \quad \cdots\cdots(3)$$

を得た。ここに，$l$, $m$, $n$ および $b$, $c$, $d$ は，the method of least squares を用いた Eqs. (4), (5) および (6) によって与えられる係数および定数である。

実測値（1965～1967年，および同3か年の平均値）を Eqs. (4), (5) および (6) に代入して，$l$, $m$, $n$ および $b$, $c$, $d$ を求めた (Table 1)。これらの値を Eqs. (1), (2) および (3) に代入して，$A_R$ と $A_L$ について各年および上記3カ年の平均値の各期における $\theta_g$ を計算する1次方程式 Eqs. (8)～(31) を組立てた。

これらの式に $\theta_a$ を代入して $\theta_g$ を計算したところ，実測値の $\theta_g$ とよく一致した (Figs. 3, 4, 5 および 6)。計算値と実測値の差を Eq.(32) によって計算すると，平均値で数%であった (Table 2, Fig. 7)。


(Received June 6, 1968)