SHALLOW-GROUNDWATER-TEMPERATURE-FLUCTUATIONS NEAR THE WATER TABLE IN THE PADDY FIELDS SHOWED BY THE FOURIER SERIES

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1. Introduction

In order to show mathematically the periodic functions in a problem of physics or engineering, the most familliar periodic functions of sin x, cos x, tan x, etc. are generally used.

The monthly average shallow-groundwater-temperature-fluctuations near the water table during a year (shown by crosses in Figs. 2, 3 and 4) also have periodic fluctuations. Therefore we applied the FOURIER series to a periodic function in order to show the fluctuation of the shallow groundwater temperature (*G. T.*) and obtained a result. The paper presented here shows the result and is the 13th report on "shallow groundwater in the downstream basin of the Aya River".

2. Theory

In order to find an equation whose value of *G.T.* varies but little from the observed ones (shown by crosses in Figs. 2, 3 and 4), we used harmonic analysis [2].

At first, we confined the problem to estimating: 1) the temperature of shallow groundwater near the water table, and 2) the average temperature for a given area.

We made the interval $[0, 2\pi]$ correspond to the length of a year (12 months) and divided it into 12 equal parts (12 months) by the points

$$\frac{0}{12}2\pi,\ \frac{1}{12}2\pi,\ \frac{2}{12}2\pi,\ \cdots\cdots\cdots,\ \frac{11}{12}2\pi. \quad \cdots\cdots\cdots (1)$$

or in degrees

$$0°, 30°, 60°, \cdots\cdots\cdots, 330° \quad \cdots\cdots\cdots (2)$$

The values of the observed *G.T.* or θ at these points (or every month) are known to be

$$\theta_0,\ \theta_1,\ \theta_2,\ \cdots\cdots\cdots, \theta_{11}. \quad \cdots\cdots\cdots (3)$$

where

$$\left.\begin{array}{l} \theta_0=\theta \text{ in January} \\ \theta_1=\theta \text{ in February} \\ \theta_2=\theta \text{ in March} \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \theta_{11}=\theta \text{ in December} \end{array}\right\} \quad \cdots\cdots\cdots (4)$$

Thus we have, for the theoretical values of *G.T.*, by using the FOURIER series [3],

$$\theta_{t_n} \approx \frac{a_0}{2} + \sum_{m=1}^{m=6} a_m \cos \frac{mn}{12} 2\pi + \sum_{m=1}^{m=5} b_m \sin \frac{mn}{12} 2\pi. \quad \cdots\cdots (5)$$

$$a_m \approx \frac{2}{12} \sum_{n=0}^{n=11} \theta_n \cos \frac{mn}{12} 2\pi. \quad \cdots\cdots (6)$$

$$b_m \approx \frac{2}{12} \sum_{n=0}^{n=11} \theta_n \sin \frac{mn}{12} 2\pi. \quad \cdots\cdots (7)$$

$$n = 0, 1, 2, \cdots\cdots 11.$$

$$m = 1, 2, \cdots\cdots 6.$$

where $\approx$ denotes approximate equality.

In this case, it is easy to see that all the factors multiplying the ordinates in Eqs. (6) and (7) reduce to

$0, \pm 1, \pm \sin 30° = \pm 0.5, \pm \sin 60° = \pm 0.866$

and it is easily verified that

$$a_0 \approx \frac{2}{12} \left\{ \theta_0 + \theta_1 + \cdots\cdots + \theta_{11} \right\} \cdots\cdots (8)$$

$$a_1 \approx \frac{2}{12} \left\{ (\theta_0 - \theta_6) + 0.866\,(\theta_1 - \theta_5 - \theta_7 + \theta_{11}) + 0.5\,(\theta_2 - \theta_4 - \theta_8 + \theta_{10}) \right\} \cdots\cdots (9)$$

$$a_2 \approx \frac{2}{12} \left\{ (\theta_0 - \theta_3 + \theta_6 - \theta_9) + 0.5\,(\theta_1 - \theta_2 - \theta_4 + \theta_5 + \theta_7 - \theta_8 - \theta_{10} + \theta_{11}) \right\} \cdots\cdots (10)$$

$$a_3 \approx \frac{2}{12} \left\{ (\theta_0 - \theta_2 + \theta_4 - \theta_6 + \theta_8 - \theta_{10}) \right\} \cdots\cdots (11)$$

$$a_4 \approx \frac{2}{12} \left\{ (\theta_0 + \theta_3 + \theta_6 + \theta_9) + 0.5\,(-\theta_1 - \theta_2 - \theta_4 - \theta_5 - \theta_7 - \theta_8 - \theta_{10} - \theta_{11}) \right\} \cdots\cdots (12)$$

$$a_5 \approx \frac{2}{12} \left\{ (\theta_0 - \theta_6) + 0.866(-\theta_1 + \theta_5 + \theta_7 - \theta_{11}) + 0.5\,(\theta_2 - \theta_4 - \theta_8 + \theta_{10}) \right\} \cdots\cdots (13)$$

$$a_6 \approx \frac{2}{12} \left\{ \theta_0 - \theta_1 + \theta_2 - \theta_3 + \theta_4 - \theta_5 + \theta_6 - \theta_7 + \theta_8 - \theta_9 + \theta_{10} - \theta_{11} \right\} \cdots\cdots (14)$$

$$b_1 \approx \frac{2}{12} \left\{ (\theta_3 - \theta_9) + 0.866(\theta_2 + \theta_4 - \theta_8 - \theta_{10}) + 0.5(\theta_1 + \theta_5 - \theta_7 - \theta_{11}) \right\} \cdots\cdots (15)$$

$$b_2 \approx \frac{2}{12} \left\{ 0.866\,(\theta_1 + \theta_2 - \theta_4 - \theta_5 + \theta_7 + \theta_8 - \theta_{10} - \theta_{11}) \right\} \cdots\cdots (16)$$

$$b_3 \approx \frac{2}{12} \left\{ \theta_1 - \theta_3 + \theta_5 - \theta_7 + \theta_9 - \theta_{11} \right\} \cdots\cdots (17)$$

$$b_4 \approx \frac{2}{12} \left\{ 0.866\,(\theta_1 - \theta_2 + \theta_4 - \theta_5 + \theta_7 - \theta_8 + \theta_{10} - \theta_{11}) \right\} \cdots\cdots (18)$$

$$b_5 \approx \frac{2}{12} \left\{ (\theta_3 - \theta_9) - 0.866\,(-\theta_2 - \theta_4 + \theta_8 + \theta_{10}) + 0.5\,(\theta_1 + \theta_5 - \theta_7 - \theta_{11}) \right\} \cdots\cdots (19)$$

For convenience of calculation in obtaining the approximate values of θt, we rewrote Eq.(5) into Eq. (20).

$$\theta_{t_n} = \frac{a_0}{2} + a_1 \cos \frac{n}{12} 2\pi + a_2 \cos \frac{2n}{12} 2\pi + \cdots\cdots + a_6 \cos \frac{6n}{12} 2\pi$$

$$+ b_1 \sin \frac{n}{12} 2\pi + b_2 \sin \frac{2n}{12} 2\pi + \cdots\cdots + b_5 \sin \frac{5n}{12} 2\pi. \cdots\cdots (20)$$

3. Data

The observed values of *G.T.* (or θ) used to calculate the values of $a_0/2$ and the FOURIER coefficients (Eqs.(7) and (8) are shown in Figs. 2, 3 and 4 (crosses)). The observed values of *G.T.* (or θ) used to estimate the theoretical values of *G.T.* (θt) by using Eq.(20) are also shown in Figs. 2, 3 and 4 (crosses). From the curves shown in these figures, we see that the values of θ fluctuate periodically.

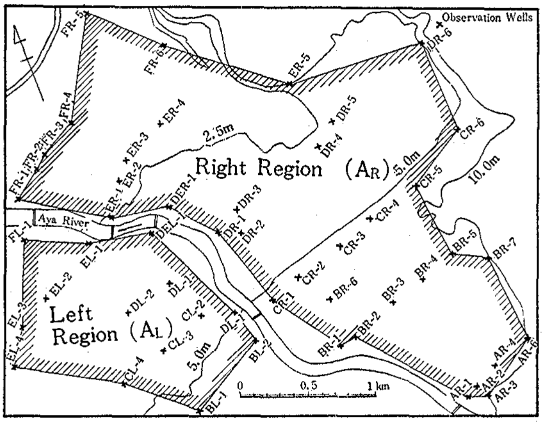

Fig. 1. Simplified map of the study area showing the observation wells (crosses) where the G.T. was measured.

We got these values of θ from data obtained from the shallow groundwater investigation that we have been carrying on since July 1964 with 51 observation wells (crosses in Fig.1). The study area has been the paddy fields in the downstream basin of the Aya River, Kagawa Prefecture. The study area, as reported in the previous paper [1], was divided into two regions, the right region or AR (492.85 *ha*) and the left region or AL (157.94 *ha*) as shown in Fig.1.

4. Results and Discussion

Using Eqs.(8) to (14) and Eqs.(15) to (19), we calculated the values of $a_0/2$ and the FOURIER coefficients (a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4 and b_5) as listed in Table 1.

Table 1 Values of the FOURIER coefficients as calculated by Eqs. (8) to (19). We used these values to obtain the theoretical values of *G.T.* shown in Figs. 2, 3 and 4.

	1965			1966			1967		
	AR	AL	AW	AR	AL	AW	AR	AL	AW
$a_0/2$	15.658	16.042	15.825	16.567	16.675	16.575	16.008	16.183	16.167
a_1	−4.108	−4.228	−4.100	−3.376	−3.677	−3.478	−4.317	−4.612	−4.500
a_2	0.807	0.850	0.825	−0.367	−0.433	−0.400	−0.008	−0.333	−0.217
a_3	−0.600	−0.333	−0.567	−0.150	0.033	−0.133	0.283	1.000	0.833
a_4	−0.533	−0.467	−0.492	−0.167	−0.150	−0.150	−0.458	−0.583	−0.587
a_5	−0.442	−0.389	−0.434	0.378	0.394	0.361	−0.016	−0.138	−0.083
a_6	−0.583	−0.350	−0.550	−0.567	−0.683	−0.550	0.217	0.567	0.533
b_1	−3.393	−3.831	−3.498	−2.822	−3.169	−2.841	−3.717	−4.206	−4.065
b_2	−0.231	0.347	−0.101	0.836	0.614	0.779	0.910	1.415	1.299
b_3	−0.250	−0.250	−0.217	−0.350	−0.350	−0.350	−0.300	−0.067	−0.133
b_4	0.318	0.029	0.245	0.201	0.289	0.231	0.159	−0.518	−0.341
b_5	0.043	0.181	0.082	0.122	−0.061	0.091	0.267	0.673	0.582

Using Eq.(20) with the values listed in Table 1, we calculated the theoretical values of *G.T.* (θt) for AR for every month during 1965. The results are shown by the open circles in Fig.2 (1965, AR).

In like manner (by using Eq.(20)), we obtained the values of θt for AL and AW during the same year. The values of θt for 1966 and 1967 were also calculated as shown in Figs. 3 and 4 respectively.

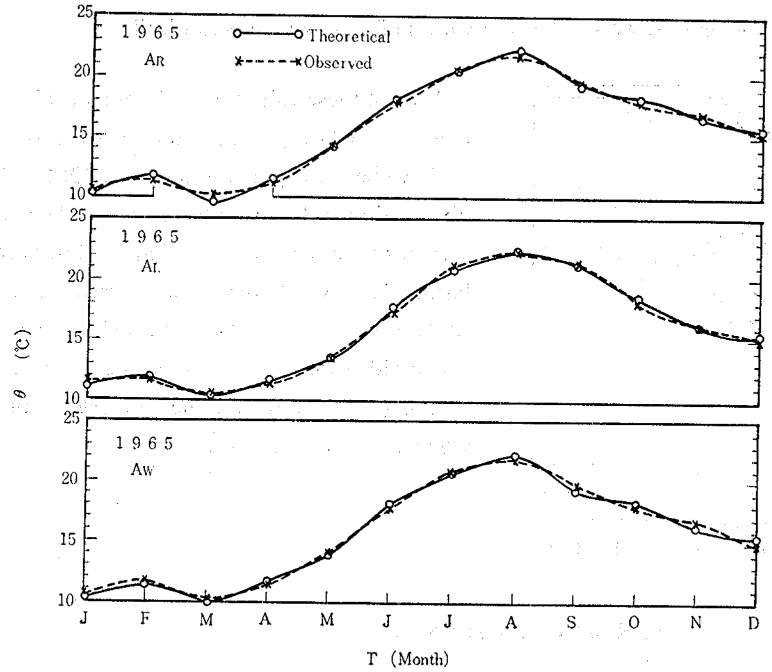

Fig. 2.
Theoretical values (open circles) and the observed values (crosses) of G.T. for AR, AL and AW during 1965.

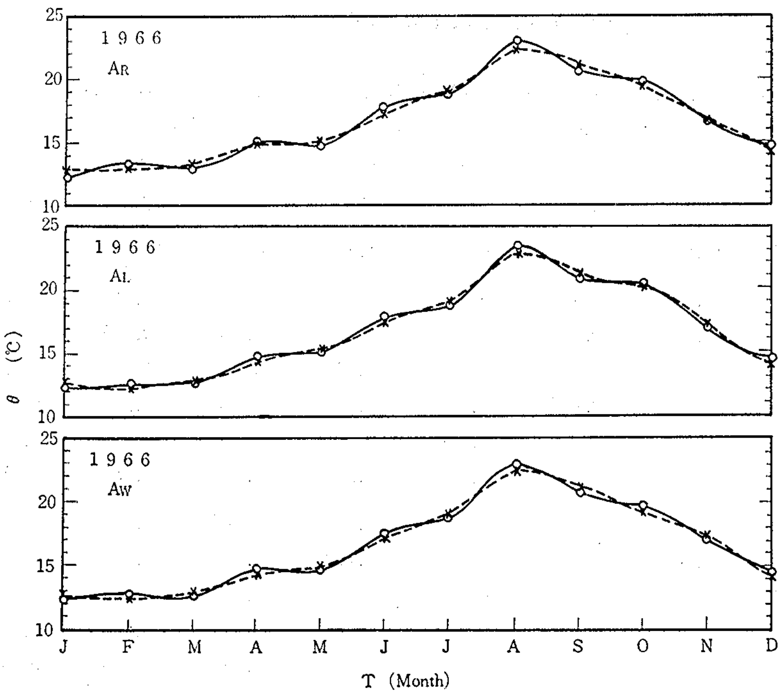

Fig. 3.
Theoretical values (open circles) and the observed values (crosses) of G.T. for AR, AL and AW during 1966.

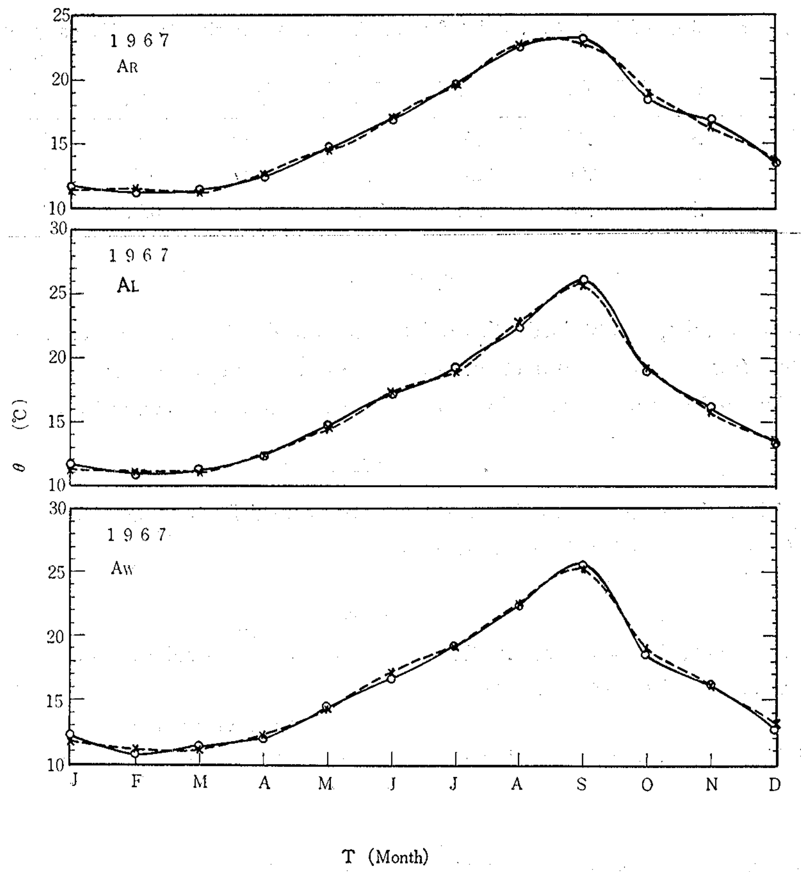

Fig. 4. Theoretical values (open circles) and the observed values (crosses) of G.T. for AR, AL and AW during 1967.

Looking at the curves shown in Figs. 2, 3 and 4, we see that the theoretical values of *G.T.* or θt (open circles) are close to the observed ones (crosses) for every region for every month throughout the entire three years.

To see how much the theoretical values of *G.T.* (θt) deviate from the observed ones (θ) in every month, we used Eq.(21) and estimated the values of the deviation (ε). Their major values are listed in Table 2.

$$\varepsilon = \left| \frac{\theta_t - \theta}{\theta} \right| \times 100\,(\%) \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (21)$$

From the values listed in Table 2, we see that the maximum values of ε or $\varepsilon_{Max.}$ are below 4.00 %; the minimum values of ε or ε_{Min} are below 1.38 %; and the mean values of ε or $\overline{\varepsilon}$ are below 2.12 % for the study area throughout the entire three years.

Table 2 The values of ε_{Max}, ε_{Min} and $\breve{\varepsilon}$ for AR, AL and AW during 1965, 1966 and 1967.

	1965			1966			1967		
	AR	AL	AW	AR	AL	AW	AR	AL	AW
ε max	2.97	1.90	3.48	4.00	3.28	2.40	0.90	2.73	2.75
ε min	1.38	0.88	0.96	1.33	1.29	1.32	0.44	1.15	1.17
$\bar{\varepsilon}$	2.05	1.33	1.86	2.01	2.12	1.88	0.76	2.00	1.92

5. Summary

Using the FOURIER series, we obtained a theoretical equation, Eq.(20), that showed the shallow groundwater temperature (monthly average) fluctuation near the water table for a year.

The values calculated by Eq.(20) were close to the observed ones as shown in Figs. 2, 3 and 4.

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FOURIER 級数で表現した水田地帯の地下水面附近の浅層地下水温の変動

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要　旨

水田地帯における地下水面附近の浅層地下水温は，1年周期の周期現象と考え，その変動の模様を *FOURIER* 級数を用いた計算式，Eq. (20)，によって表現しようと試みた。

θ_t を（計算値）地下水温（°C）とすると，

$$\theta_{\underset{n}{t}} = \frac{a_0}{2} + a_1 \cos \frac{n}{12} 2\pi + a_2 \cos \frac{2n}{12} 2\pi + \cdots\cdots + a_6 \cos \frac{6n}{12} 2\pi$$

$$+ b_1 \sin \frac{n}{12} 2\pi + b_2 \sin \frac{2n}{12} 2\pi + \cdots\cdots + b_5 \sin \frac{5n}{12} 2\pi \cdots\cdots (20)$$

$n=0,\ 1,\ 2,\ \cdots\cdots,\ 11$（$n=0$ は1月，$n=1$ は2月，……，$n=11$は12月に対応する）

ここに，$\frac{a_0}{2}$は Eq. (8) によって，また *FOURIER* 係数 a_1, a_2, ……, a_6 および b_1, b_2, ……, b_5は，それぞれ Eqs. (9)～(19) によって与えられる。

実測値（θ, °C）（1965～1967年）を各式に代入し θ_t を計算した結果，上記の3カ年間を通じて，θ_t は θ とよく一致し (Figs. 2, 3および4)，その度合は，Eq. (21) によって積算すると，最大値で4％以下，平均値で2.12％以下であった。

計算に用いた（ある月の）実測値は，研究地域（香川県綾川下流域の水田地帯，650.79*ha*）内の51測点（浅井戸，Fig. 1）で，（その月に）観測したm個（$m \leqq 51$）の観測値の算術平均（地域全体の平均）値である。

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