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BEST LINEAR INVARIANT ESTIMATES OF PARAMETERS OF THE LARGEST EXTREME-VALUE DISTRIBUTION FROM COMPLETE AND FROM SINGLY CENSORED SAMPLES AND ITS APPLICATIONS

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第一種極大値漸近分布母数の最良線形不変推定量とその応用

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The problem of choosing the best estimators to estimate the parameters for the Type I asymptotic distribution of the largest extreme based on the first m order statistics of a sample of size n is considered, and the best linear invariant (**BLI**) estimators, which are best among linear estimators with mean squared error invariant under transformations, are presented and discussed.

The weights for obtaining the **BLI** estimates and the mean squared errors of the estimators are given in Table I for $2\leq m\leq 15$, $2\leq m\leq n$. Use of the **BLI** weights is illustrated by a detailed discussion of the case that a sample of size n can be assumed to be drawn from the Type I asymptotic distribution of the largest extreme.

The **BLI** estimators, which have uniformly smaller mean squared errors than the Gauss-Markov best linear unbiased (**BLU**) estimators and which are simple linear functions of the **BLU** estimators, are found to yield estimates superior to others which have been proposed to deal with this problem.

第一種極大値漸近分布に従う n 個の標本をランダムに抽出するとき，最初の m 個の順序統計量から分布母数を推定する方法について述べる．推定量の選択基準として最小平均2乗誤差を考える．HARTER & MOORE および MANN の結果から最良線形不変推定量は最小平均2乗誤差をもつことがわかる．最良線形不変推定量は最良線形不偏推定量より一様に小さい平均2乗誤差をもち，変換に対して平均2乗誤差は不変である．

最良線形不変推定値を求めるための重みと平均2乗誤差を $2\leq m\leq 15$, $2\leq m\leq n$ について表にした．その重みの使用方法について降雨データで説明した．

1. Introduction

Assume that a sample of size n is randomly drawn from the Type I asymptotic distribution of the largest extreme whose c.d.f. is

$$F(x)=\exp[-\exp(-(x-u)/b)],\quad -\infty<x<\infty,\quad b>0. \tag{1}$$

The parameters u and b are location and scale parameters and unknown. The parameter x_P, the $100P$th percentile of the distribution, is equal to $u-b\ln\ln(1/P)$. In a meterological situation estimation of x_P and hence the return period $T(x)=1/\bar{F}(x)=1/(1-\exp[-\exp(-(x-u)/b)])$ for the given model will be of particular interest. Assume futhermore that one or more of the sample are spurious and ought to be removed from estimation. In the present context this is equivalent to assuming the n-m largest observations are censored.

The estimation problem which we shall be concered with is to choose the "best" estimators for the largest extreme-value parameters based on the first m order statistics of a sample of size n. Various estimators have been proposed and discussed in the past (for example, see GUMBEL[1]), but none of

these are appropriate for encored samples. The main point of this paper will be to introduce the estimators, that is, the best linear invariant (**BLI**) estimators, to overcome some of the drawbacks of the estimators GUMBEL discussed.

By analogy with the smallest extreme-value distribution, we shall employ the **BLI** estimators, proposed by MANN[4)-8)]. That is, from the point of view of mean squared error, Harter and MOORE[2)] have pointed out that the maximum-likelihood estimators and the **BLI** estimators are two best of others. The linear estimation procedures, of course, are more convenient to implememt.

2. Moments of Order Statistics

Suppose that $X_{1,n}\leq X_{2,n}\leq\cdots\leq X_{m,n}$ is a first m of n ordered sample from the Type I asymptotic distribution of the largest extreme-value with location parameter u and scale parameter b. Following Lieblein[3)], we shall consider the reduced ordered sample $Y_{i,n}=(X_{i,n}-u)/b$, $i=1,2,\cdots,m$. The density function of the ith ordered statistic $Y_{i,n}$ is

$$f_{Y_{i,n}}(x)=\frac{n!}{(i-1)!\,(n-i)!}e^{-(i-1)e^{-x}}(1-e^{-e^{-x}})^{n-i}e^{-x-e^{-x}},\quad -\infty<x>\infty. \tag{2}$$

The joint d. f. of the ith and jth ordered statistics $Y_{i,n}$ and $Y_{j,n}$ is

$$f_{Y_{i,n}Y_{j,n}}(x,y)=\frac{n!}{(i-1)!\,(j-i-1)!\,(n-j)!}e^{-y-e^{-y}}e^{-x-ie^{-x}}(e^{-e^{-y}}-e^{-e^{-x}})^{j-i-1}(1-e^{-e^{-y}})^{n-j},$$
$$-\infty<x\leq y<\infty. \tag{3}$$

Thus, we obtain

$$E(y_i{}^k)=\frac{n!}{(i-1)!\,(n-i)!}\sum_{r=0}^{n-i}(-1)^r\binom{n-i}{r}g_k(i+r), \tag{4}$$

and

$$E(y_iy_j)=\frac{n!}{(i-1)!\,(j-i-1)!\,(n-j)!}\sum_{r=0}^{j-i-1}\sum_{s=0}^{n-j}(-1)^{r+s}\binom{j-i-1}{r}\binom{n-j}{s}\phi(i+r,j-i-r+s), \tag{5}$$

where

$$g_k(c)=\int_{-\infty}^{\infty}x^ke^{-x-ce^{-x}}dx, \tag{6}$$

and

$$\phi(c_1,c_2)=\int_{-\infty}^{\infty}\int_{-\infty}^{y}xye^{-x-c_1e^{-x}}e^{-y-c_2e^{-y}}dx\,dy,\qquad c_1,c_2>0. \tag{7}$$

It is known that

$$g_1(c)=\frac{1}{c}(\gamma+\log c), \tag{8}$$

$$g_2(c)=\frac{1}{c}\left(\frac{\pi^2}{6}+(\gamma+\log c)^2\right), \tag{9}$$

$$\phi(c,c)=\frac{1}{2c^2}(\gamma+\log c)^2, \tag{10}$$

$$\phi(c_1,c_2)=\frac{1}{2c_1c_2}\left\{(c_2-c_1)g_2(c_1+c_2)+c_1{}^2\{g_1(c_1)\}^2+2L\left(1+\frac{c_1}{c_2}\right)-\left\{\log\frac{c_1}{c_2}\right\}^2-\frac{\pi^2}{6}\right\},\qquad c_1<c_2 \tag{11}$$

$$\phi(c_1,c_2)=\frac{1}{2c_1c_2}\left\{(c_2-c_1)g_2(c_1+c_2)+c_1{}^2\{g_1(c_1)\}^2-2L\left(1+\frac{c_2}{c_1}\right)+\frac{\pi^2}{6}\right\},\qquad c_1>c_2 \tag{12}$$

where γ is Euler's constant, 0.577215664901532$\cdots$ and

$$L(1+x)=\int_1^{1+x}\frac{\log z}{1-z}dz=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n^2}$$

is Spence's integral (see POWELL[9)]).

3. Linear Estimation

We shall first consider the method for constructing the best linear unbiased estimators for u, b, and X_P. Let $X^*_{P,m,n}$ be an unbiased linear estimator for $X_{P,m,n}$, that is, let

$$X^*_{P,m,n}=u^*_{m,n}-b^*_{m,n}\ \ln\ \ln(1/P)$$
$$=\sum_{i=1}^{m}\{a_{i,m,n}-\ln\ \ln(1/P)c_{i,m,n}\}X_{i,n}. \tag{13}$$

For the expectation of $X^*_{P,m,n}$ we have from the definition

$$E(X^*_{P,m,n})=E(\sum_{i=1}^{m}\{a_{i,m,n}-\ln\ \ln(1/P)c_{i,m,n}\}X_{i,n})$$
$$=X_{P,m,n}$$
$$=u_{m,n}-b_{m,n}\ln\ln(1/P), \tag{14}$$

or equivalently

$$E(u^*_{m,n})=E(\sum_{i=1}^{m}a_{i,m,n}X_{i,n})=u_{m,n}, \tag{15}$$

and

$$E(b^*_{m,n})=E(\sum_{i=1}^{m}c_{i,m,n}X_{i,n})=b_{m,n}. \tag{16}$$

Since $X_{i,n}=(Y_{i,n}-u)/b$, (15) and (16) reduce to

$$E(u^*_{m,n})=\sum_{i=1}^{m}a_{i,m,n}E(u_{m,n}+b_{m,n}Y_{i,n}) \tag{17}$$

and

$$E(b^*_{m,n})=\sum_{i=1}^{m}c_{i,m,n}E(u_{m,n}+b_{m,n}Y_{i,n}). \tag{18}$$

Therefore we have the following result:

$$E(u^*_{m,n})=u_{m,n}\text{iff}\ \sum_{i=1}^{m}a_{i,m,n}=1\ \text{and}\ \sum_{i=1}^{m}a_{i,m,n}E(Y_{i,n})=0 \tag{19}$$

and

$$E(b^*_{m,n})=b_{m,n}\ \text{iff}\ \sum_{i=1}^{m}c_{i,m,n}=0\ \text{and}\ \sum_{i=1}^{m}c_{i,m,n}E(Y_{i,n})=1. \tag{20}$$

Then we should like to find $\{a_{i,m,n}\}$ and $\{c_{i,m,n}\}$ so as to minimize the variances of $u^*_{m,n}$ and $b^*_{m,n}$

$$\sum_{i=1}^{m}\sum_{j=1}^{m}a_{i,m,n}a_{j,m,n}\mathrm{Cov}(X_{i,n},X_{j,n}) \tag{21}$$

and

$$\sum_{i=1}^{m}\sum_{j=1}^{m}c_{i,m,n}c_{j,m,n}\mathrm{Cov}(X_{i,n},X_{j,n}), \tag{22}$$

respectively. We see that $\mathrm{Cov}(X_{i,n},X_{j,n})$ is equal to $b^2\ \mathrm{Cov}(Y_{i,n},X_{j,n})$. Hence, minimizing (21) and (22) with respect to $a_{i,m,n}$ and $c_{i,m,n}$, subject to the conditions (19) and (20), is equivalent to minimizing the following forms:

$$\sum_{i=1}^{m}\sum_{j=1}^{m}a_{i,m,n}a_{j,m,n}\mathrm{Cov}(Y_{i,n},Y_{j,n})+\lambda_1\sum_{i=1}^{m}a_{i,m,n}+\lambda_2\sum_{i=1}^{m}a_{i,m,n}E(Y_{i,n}) \tag{23}$$

and

$$\sum_{i=1}^{m}\sum_{i=1}^{m}c_{i,m,n}c_{j,m,n}\mathrm{Cov}(Y_{i,n},Y_{j,n})+\lambda_3\sum_{i=1}^{m}c_{i,m,n}+\lambda_4\sum_{i=1}^{m}c_{i,m,n}E(Y_{i,n}), \tag{24}$$

respectively, where λ_s are Lagrange multipliers. It will be convenient to let $a_i=a_{i,m,n}$, $c_i=c_{i,m,n}$, $\sigma_{i,j}=\mathrm{Cov}(Y_{i,n},Y_{j,n})$, and $E_i=E(Y_{i,n})$, $i,j=1,2,\cdots,m$. Defferentiating (23) and (24) with respect to $a_{i,m,n}$ and $c_{i,m,n}$ we obtain $(m+2)$ linear simultaneous equations

Table 1. Weights for oftaining best linear invariant estimates of parameters of the largest extreme-value distribution.

$A(N,M,I)$: Weights for estimating u
$C(N,M,I)$: Weights for estimating b
$b^2E(LU)$: Mean squared error for u
$b^2E(CP)$: $E(u-u)(\tilde{b}-b)$
$b^2E(LB)$: Mean squared error for $\tilde{b}$

N	M	I	A(N,M,I)	C(N,M,I)
2	2	1	0.889269	-0.421383
		2	0.110731	0.421383
E(LU)			0.6571300	
E(CP)			-0.0375742	
E(LB)				0.4158392
3	2	1	0.603352	-0.690920
		2	0.396648	0.690920
E(LU)			0.4148771	
E(CP)			0.0612878	
E(LB)				0.4037041
3	3	1	0.667936	-0.468904
		2	0.251001	0.190239
		3	0.081063	0.278666
E(LU)			0.4024074	
E(CP)			0.0184217	
E(LB)				0.2563462
4	2	1	0.340634	-0.879965
		2	0.659366	0.879965
E(LU)			0.3371736	
E(CP)			0.1254102	
E(LB)				0.4019794
4	3	1	0.483201	-0.593557
		2	0.270739	0.099235
		3	0.246060	0.494322
E(LU)			0.2988436	
E(CP)			0.0484073	
E(LB)				0.2472848
4	4	1	0.526813	-0.455910
		2	0.261510	0.070109
		3	0.147340	0.182749
		4	0.064336	0.203052
E(LU)			0.2924765	
E(CP)			0.0283121	
E(LB)				0.1838619
5	2	1	0.112593	-1.024151
		2	0.887407	1.024151
E(LU)			0.3127041	
E(CP)			0.1725667	
E(LB)				0.4026408
5	3	1	0.324417	-0.689924
		2	0.255634	0.027310
		3	0.419949	0.662615
E(LU)			0.2492961	
E(CP)			0.0725185	
E(LB)				0.2447802
5	4	1	0.402049	-0.525419
		2	0.247811	0.010733
		3	0.173070	0.139467
		4	0.177069	0.375219
E(LU)			0.2342760	
E(CP)			0.0406902	
E(LB)				0.1773343
5	5	1	0.433593	-0.431259
		2	0.246092	0.005600
		3	0.163808	0.111820
		4	0.103531	0.155707
		5	0.052975	0.158131
E(LU)			0.2304050	
E(CP)			0.0291352	
E(LB)				0.1428429
6	2	1	-0.086117	-1.140225
		2	1.086117	1.140225
E(LU)			0.3099333	
E(CP)			0.2095161	
E(LB)				0.4039368
6	3	1	0.186635	-0.767957
		2	0.227525	-0.031630
		3	0.585840	0.799587
E(LU)			0.2242859	
E(CP)			0.0926197	
E(LB)				0.2443900
6	4	1	0.294288	-0.583222
		2	0.224688	-0.036498
		3	0.179161	0.101713
		4	0.301863	0.518007
E(LU)			0.2006959	
E(CP)			0.0521384	
E(LB)				0.1749228
6	5	1	0.344243	-0.474513
		2	0.225354	-0.035049
		3	0.168128	0.077704
		4	0.124635	0.132333
		5	0.137640	0.299524
E(LU)			0.1928985	
E(CP)			0.0351703	
E(LB)				0.1379977
6	6	1	0.368179	-0.405733
		2	0.226486	-0.031796
		3	0.163591	0.064666
		4	0.117541	0.111951
		5	0.079377	0.132102
		6	0.044826	0.128810
E(LU)			0.1903043	
E(CP)			0.0277157	
E(LB)				0.1165767

Table of Weights (continued)

N	M	I	A(N,M,I)	C(N,M,I)
7	2	1	-0.261396	-1.237150
		2	1.261396	1.237150
E(LU)			0.3171154	
E(CP)			0.2396703	
E(LB)				0.4053668
7	3	1	0.065401	-0.833308
		2	0.194342	-0.081469
		3	0.740257	0.914777
E(LU)			0.2119701	
E(CP)			0.1097364	
E(LB)				0.2448002
7	4	1	0.199672	-0.632201
		2	0.198107	-0.075830
		3	0.175548	0.068975
		4	0.426673	0.639056
E(LU)			0.1804631	
E(CP)			0.0625464	
E(LB)				0.1741207
7	5	1	0.265863	-0.512564
		2	0.202305	-0.068242
		3	0.164571	0.049136
		4	0.134018	0.110097
		5	0.233242	0.421574
E(LU)			0.1687550	
E(CP)			0.0413846	
E(LB)				0.1358718
7	6	1	0.301107	-0.434663
		2	0.205584	-0.060994
		3	0.159741	0.038459
		4	0.125255	0.090726
		5	0.096117	0.118479
		6	0.112196	0.247993
E(LU)			0.1640448	
E(CP)			0.0309734	
E(LB)				0.1128592
7	7	1	0.319934	-0.382022
		2	0.207825	-0.054727
		3	0.157657	0.032632
		4	0.120971	0.078748
		5	0.090785	0.103569
		6	0.064086	0.113479
		7	0.038743	0.108323
E(LU)			0.1621907	
E(CP)			0.0257894	
E(LB)				0.0983650
8	2	1	-0.417919	-1.320235
		2	1.417919	1.320235
E(LU)			0.3291217	
E(CP)			0.2649998	
E(LB)				0.4067683
8	3	1	-0.042672	-0.889418
		2	0.159429	-0.124622
		3	0.883243	1.014040
E(LU)			0.2067962	
E(CP)			0.1245594	
E(LB)				0.2455305
8	4	1	0.115449	-0.674513
		2	0.170501	-0.109575
		3	0.166736	0.040222
		4	0.547314	0.743865
E(LU)			0.1680944	
E(CP)			0.0719591	
E(LB)				0.1740404
8	5	1	0.196117	-0.545995
		2	0.178758	-0.096419
		3	0.156882	0.024522
		4	0.136804	0.089852
		5	0.331440	0.528040
E(LU)			0.1526949	
E(CP)			0.0474250	
E(LB)				0.1349534
8	6	1	0.241337	-0.461288
		2	0.184594	-0.085487
		3	0.152571	0.016447
		4	0.127327	0.072100
		5	0.105284	0.104401
		6	0.188887	0.353826
E(LU)			0.1458762	
E(CP)			0.0346522	
E(LB)				0.1110272
8	7	1	0.267713	-0.402388
		2	0.188606	-0.076530
		3	0.150687	0.012240
		4	0.122344	0.060974
		5	0.098663	0.089617
		6	0.077527	0.105155
		7	0.094460	0.210933
E(LU)			0.1427482	
E(CP)			0.0276673	
E(LB)				0.0954297
8	8	1	0.282943	-0.360675
		2	0.191236	-0.069325
		3	0.149934	0.010179
		4	0.119768	0.053918
		5	0.095062	0.079752
		6	0.073452	0.093994
		7	0.053552	0.098886
		8	0.034052	0.093270
E(LU)			0.1413603	
E(CP)			0.0238656	
E(LB)				0.0850168

Table of Weights (continued)

	N	M	I	A(N,M,I)	C(N,M,I)
	9	2	1	-0.559212	-1.392869
			2	1.559212	1.392869
E(LU)				0.3434708	
E(CP)				0.2867426	
E(LB)					0.4080889
	9	3	1	-0.140098	-0.938516
			2	0.124324	-0.162666
			3	1.015774	1.101182
E(LU)				0.2058750	
E(CP)				0.1375775	
E(LB)					0.2463818
	9	4	1	0.039607	-0.711669
			2	0.142958	-0.139143
			3	0.155019	0.014629
			4	0.662416	0.836183
E(LU)				0.1606442	
E(CP)				0.0804816	
E(LB)					0.1743083
	9	5	1	0.133332	-0.575622
			2	0.155494	-0.120947
			3	0.146922	0.002877
			4	0.135606	0.071485
			5	0.428647	0.622208
E(LU)				0.1418166	
E(CP)				0.0531522	
E(LB)					0.1346380
	9	6	1	0.187484	-0.485391
			2	0.164068	-0.106660
			3	0.143614	-0.002636
			4	0.126044	0.055554
			5	0.109842	0.091003
			6	0.268947	0.448130
E(LU)				0.1329828	
E(CP)				0.0384330	
E(LB)					0.1101124
	9	7	1	0.220525	-0.421846
			2	0.169992	-0.095267
			3	0.142328	-0.005110
			4	0.120872	0.045605
			5	0.102475	0.076835
			6	0.085747	0.095795
			7	0.158062	0.303988
E(LU)				0.1285876	
E(CP)				0.0299801	
E(LB)					0.0938554

	N	M	I	A(N,M,I)	C(N,M,I)
	9	8	1	0.241100	-0.375569
			2	0.174061	-0.086116
			3	0.141938	-0.005987
			4	0.118033	0.039220
			5	0.098199	0.067216
			6	0.080703	0.084449
			7	0.064549	0.093660
			8	0.081418	0.183125
E(LU)				0.1263713	
E(CP)				0.0249950	
E(LB)					0.0826431
	9	9	1	0.253697	-0.341614
			2	0.176764	-0.078828
			3	0.141932	-0.006001
			4	0.116517	0.035136
			5	0.095769	0.060667
			6	0.077742	0.076470
			7	0.061368	0.085084
			8	0.045872	0.087308
			9	0.030338	0.081777
E(LU)				0.1252952	
E(CP)				0.0220944	
E(LB)					0.0748243
	10	2	1	-0.687940	-1.457344
			2	1.687940	1.457344
E(LU)				0.3588770	
E(CP)				0.3057248	
E(LB)					0.4093150
	10	3	1	-0.228764	-0.982122
			2	0.089762	-0.196681
			3	1.139002	1.178803
E(LU)				0.2075851	
E(CP)				0.1491462	
E(LB)					0.2472649
	10	4	1	-0.029350	-0.744744
			2	0.115984	-0.165466
			3	0.141657	-0.008416
			4	0.771709	0.918626
E(LU)				0.1564097	
E(CP)				0.0882280	
E(LB)					0.1747491
	10	5	1	0.076269	-0.602138
			2	0.132857	-0.142686
			3	0.135703	-0.016455
			4	0.131891	0.054745
			5	0.523281	0.706534
E(LU)				0.1344115	
E(CP)				0.0585261	
E(LB)					0.1346456

Table of Weights (continued)

N	M	I	A(N,M,I)	C(N,M,I)
10	6	1	0.138516	-0.507218
		2	0.144231	-0.125341
		3	0.133690	-0.019524
		4	0.122665	0.040676
		5	0.111417	0.078488
		6	0.349481	0.532919
E(LU)			0.1236801	
E(CP)			0.0421619	
E(LB)				0.1096920
10	7	1	0.177542	-0.439914
		2	0.152120	-0.111736
		3	0.133242	-0.020297
		4	0.117636	0.032004
		5	0.103687	0.065157
		6	0.090623	0.086501
		7	0.225149	0.388285
E(LU)			0.1180645	
E(CP)			0.0324774	
E(LB)				0.0929905
10	8	1	0.202839	-0.390262
		2	0.157663	-0.100856
		3	0.133421	-0.019947
		4	0.114823	0.026482
		5	0.099060	0.056075
		6	0.084938	0.075343
		7	0.071762	0.087218
		8	0.135495	0.265947
E(LU)			0.1150261	
E(CP)			0.0265136	
E(LB)				0.0812848
10	9	1	0.219391	-0.352833
		2	0.161542	-0.092085
		3	0.133817	-0.019050
		4	0.113254	0.022934
		5	0.096274	0.049774
		6	0.081405	0.067354
		7	0.067844	0.078358
		8	0.055033	0.083994
		9	0.071442	0.161555
E(LU)			0.1133798	
E(CP)			0.0227907	
E(LB)				0.0728661
10	10	1	0.230001	-0.324597
		2	0.164178	-0.085070
		3	0.134239	-0.017927
		4	0.112414	0.020698
		5	0.094638	0.045420
		6	0.079263	0.061652
		7	0.065408	0.071876
		8	0.052496	0.077242
		9	0.040034	0.077971
		10	0.027331	0.072734
E(LU)			0.1125222	
E(CP)			0.0205085	
E(LB)				0.0667925

N	M	I	A(N,M,I)	C(N,M,I)
11	2	1	-0.806147	-1.515278
		2	1.806147	1.515278
E(LU)			0.3746428	
E(CP)			0.3225233	
E(LB)				0.4104481
11	3	1	-0.310108	-1.021318
		2	0.056096	-0.227438
		3	1.254012	1.248756
E(LU)			0.2109638	
E(CP)			0.1595302	
E(LB)				0.2481381
11	4	1	-0.092561	-0.774516
		2	0.089809	-0.189192
		3	0.127385	-0.029373
		4	0.875366	0.993081
E(LU)			0.1543494	
E(CP)			0.0953027	
E(LB)				0.1752736
11	5	1	0.023982	-0.626090
		2	0.110991	-0.162215
		3	0.123807	-0.033930
		4	0.126539	0.039394
		5	0.614682	0.782842
E(LU)			0.1294168	
E(CP)			0.0635492	
E(LB)				0.1348332
11	6	1	0.093638	-0.527075
		2	0.125162	-0.142071
		3	0.123270	-0.034694
		4	0.117933	0.027161
		5	0.110979	0.066837
		6	0.429019	0.609843
E(LU)			0.1169071	
E(CP)			0.0457668	
E(LB)				0.1095560
11	7	1	0.138108	-0.456584
		2	0.135021	-0.126444
		3	0.123807	-0.033842
		4	0.113273	0.019774
		5	0.103159	0.054442
		6	0.093232	0.077581
		7	0.293399	0.465072
E(LU)			0.1101313	
E(CP)			0.0350264	
E(LB)				0.0925311

Table of Weights (continued)

N	M	I	A(N,M,I)	C(N,M,I)
11	8	1	0.167657	-0.404209
		2	0.142039	-0.114004
		3	0.124680	-0.032295
		4	0.110676	0.015171
		5	0.098408	0.046021
		6	0.087151	0.066802
		7	0.076463	0.080548
		8	0.192926	0.341966
E(LU)			0.1062973	
E(CP)			0.0282306	
E(LB)				0.0804853
11	9	1	0.187703	-0.364215
		2	0.147083	-0.103942
		3	0.125587	-0.030485
		4	0.109225	0.012276
		5	0.095468	0.040155
		6	0.083261	0.059041
		7	0.072019	0.071683
		8	0.061341	0.079450
		9	0.118313	0.236038
E(LU)			0.1040867	
E(CP)			0.0238202	
E(LB)				0.0716866
11	10	1	0.201340	-0.333249
		2	0.150689	-0.095753
		3	0.126402	-0.028634
		4	0.108435	0.010481
		5	0.093651	0.036029
		6	0.080772	0.053388
		7	0.069113	0.065084
		8	0.058235	0.072398
		9	0.047788	0.075885
		10	0.063575	0.144371
E(LU)			0.1028191	
E(CP)			0.0209419	
E(LB)				0.0651501
11	11	1	0.210412	-0.309357
		2	0.153197	-0.089148
		3	0.127068	-0.026879
		4	0.108034	0.009427
		5	0.092560	0.033156
		6	0.079220	0.049300
		7	0.067261	0.060207
		8	0.056215	0.067076
		9	0.045727	0.070456
		10	0.035456	0.070318
		11	0.024850	0.065444
E(LU)			0.1021204	
E(CP)			0.0191016	
E(LB)				0.0603037
12	2	1	-0.915425	-1.567854
		2	1.915425	1.567854
E(LU)			0.3903802	
E(CP)			0.3375554	
E(LB)				0.4114947
12	3	1	-0.385249	-1.056896
		2	0.023484	-0.255508
		3	1.361764	1.312403
E(LU)			0.2154135	
E(CP)			0.1689309	
E(LB)				0.2489824
12	4	1	-0.150907	-0.801566
		2	0.064525	-0.210791
		3	0.112651	-0.048586
		4	0.973732	1.060943
E(LU)			0.1537971	
E(CP)			0.1017959	
E(LB)				0.1758346
12	5	1	-0.024261	-0.647905
		2	0.089944	-0.179951
		3	0.111584	-0.049880
		4	0.120104	0.025230
		5	0.702630	0.852505
E(LU)			0.1261418	
E(CP)			0.0682415	
E(LB)				0.1351229
12	6	1	0.052230	-0.545246
		2	0.106874	-0.157228
		3	0.112631	-0.048475
		4	0.112314	0.014776
		5	0.109141	0.055978
		6	0.506810	0.680195
E(LU)			0.1119669	
E(CP)			0.0492172	
E(LB)				0.1095901
12	7	1	0.101701	-0.471971
		2	0.118680	-0.139742
		3	0.114244	-0.046085
		4	0.108178	0.008650
		5	0.101431	0.044558
		6	0.094243	0.069113
		7	0.361523	0.535477
E(LU)			0.1040936	
E(CP)			0.0375556	
E(LB)				0.0923172
12	8	1	0.135127	-0.417291
		2	0.127156	-0.125877
		3	0.115889	-0.043395
		4	0.105927	0.004968
		5	0.096717	0.036847
		6	0.087950	0.058818
		7	0.079403	0.073976
		8	0.251832	0.411954
E(LU)			0.0994983	
E(CP)			0.0300384	
E(LB)				0.0800203

Table of Weights (continued)

N	M	I	A(N,M,I)	C(N,M,I)
12	9	1	0.158328	-0.375232
		2	0.133345	-0.114657
		3	0.117374	-0.040702
		4	0.104708	0.002757
		5	0.093764	0.031493
		6	0.083857	0.051399
		7	0.074597	0.065264
		8	0.065706	0.074538
		9	0.168321	0.305140
E(LU)			0.0967398	
E(CP)			0.0250376	
E(LB)				0.0709548
12	10	1	0.174640	-0.342249
		2	0.137891	-0.105465
		3	0.118640	-0.038143
		4	0.104072	0.001472
		5	0.091897	0.027719
		6	0.081166	0.045958
		7	0.071369	0.058737
		8	0.062173	0.067395
		9	0.053324	0.072625
		10	0.104827	0.211951
E(LU)			0.0950675	
E(CP)			0.0216565	
E(LB)				0.0641184
12	11	1	0.186091	-0.316157
		2	0.141208	-0.097906
		3	0.119672	-0.035791
		4	0.103773	0.000789
		5	0.090727	0.025052
		6	0.079404	0.041942
		7	0.069208	0.053814
		8	0.059769	0.061917
		9	0.050816	0.066911
		10	0.042110	0.069046
		11	0.057221	0.130382
E(LU)			0.0940637	
E(CP)			0.0193692	
E(LB)				0.0589066
12	12	1	0.193947	-0.295648
		2	0.143566	-0.091751
		3	0.120475	-0.033696
		4	0.103664	0.000505
		5	0.090017	0.023199
		6	0.078281	0.039010
		7	0.067800	0.050137
		8	0.058175	0.057754
		9	0.049122	0.062489
		10	0.040408	0.064601
		11	0.031776	0.063952
		12	0.022771	0.059449
E(LU)			0.0934839	
E(CP)			0.0178554	
E(LB)				0.0549544

N	M	I	A(N,M,I)	C(N,M,I)
13	2	1	-1.017034	-1.615964
		2	2.017034	1.615964
E(LU)			0.4058717	
E(CP)			0.3511326	
E(LB)				0.4124629
13	3	1	-0.455068	-1.089454
		2	-0.008020	-0.281321
		3	1.463087	1.370776
E(LU)			0.2205508	
E(CP)			0.1775043	
E(LB)				0.2497895
13	4	1	-0.205086	-0.826337
		2	0.040155	-0.230616
		3	0.097732	-0.066323
		4	1.067200	1.123276
E(LU)			0.1543112	
E(CP)			0.1077841	
E(LB)				0.1764058
13	5	1	-0.069038	-0.667915
		2	0.069712	-0.196197
		3	0.099251	-0.064555
		4	0.112948	0.012089
		5	0.787127	0.916577
E(LU)			0.1241215	
E(CP)			0.0726294	
E(LB)				0.1354697
13	6	1	0.013799	-0.561968
		2	0.089345	-0.171087
		3	0.101945	-0.061109
		4	0.106112	0.003346
		5	0.106310	0.045827
		6	0.582489	0.744991
E(LU)			0.1083854	
E(CP)			0.0525033	
E(LB)				0.1097287
13	7	1	0.067899	-0.486213
		2	0.103060	-0.151882
		3	0.104688	-0.057268
		4	0.102608	-0.001562
		5	0.098855	0.035388
		6	0.094096	0.061102
		7	0.428795	0.600435
E(LU)			0.0994755	
E(CP)			0.0400268	
E(LB)				0.0922580

Table of Weights (continued)

N	M	I	A(N,M,I)	C(N,M,I)
13	8	1	0.104898	-0.429521
		2	0.112965	-0.136705
		3	0.107152	-0.053493
		4	0.100793	-0.004343
		5	0.094296	0.028403
		6	0.087729	0.051345
		7	0.081062	0.067621
		8	0.311106	0.476693
E(LU)			0.0941549	
E(CP)			0.0318743	
E(LB)				0.0797663
13	9	1	0.130981	-0.385718
		2	0.120274	-0.124430
		3	0.109257	-0.049957
		4	0.099883	-0.005871
		5	0.091431	0.023591
		6	0.083551	0.044329
		7	0.076021	0.059156
		8	0.068675	0.069573
		9	0.219926	0.369327
E(LU)			0.0908695	
E(CP)			0.0263571	
E(LB)				0.0705012
13	10	1	0.149713	-0.351114
		2	0.125734	-0.114345
		3	0.111008	-0.046722
		4	0.099473	-0.006628
		5	0.089606	0.020220
		6	0.080761	0.039176
		7	0.072581	0.052800
		8	0.064828	0.062467
		9	0.057319	0.068937
		10	0.148976	0.275210
E(LU)			0.0888036	
E(CP)			0.0225407	
E(LB)				0.0634510
13	11	1	0.163269	-0.323395
		2	0.129824	-0.105982
		3	0.112436	-0.043803
		4	0.099339	-0.006901
		5	0.088443	0.017842
		6	0.078888	0.035346
		7	0.070218	0.047968
		8	0.062145	0.056979
		9	0.054463	0.063096
		10	0.046995	0.066682
		11	0.093981	0.192167
E(LU)			0.0874993	
E(CP)			0.0198738	
E(LB)				0.0579979
13	12	1	0.173037	-0.301077
		2	0.132864	-0.099034
		3	0.113573	-0.041206
		4	0.099354	-0.006867
		5	0.087714	0.016175
		6	0.077640	0.032494
		7	0.068605	0.044282
		8	0.060284	0.052728
		9	0.052454	0.058507
		10	0.044935	0.061977
		11	0.037554	0.063233
		12	0.051986	0.118787
E(LU)			0.0866860	
E(CP)			0.0180154	
E(LB)				0.0537515
13	13	1	0.179913	-0.283257
		2	0.135068	-0.093324
		3	0.114446	-0.038943
		4	0.099440	-0.006644
		5	0.087274	0.015037
		6	0.076831	0.030398
		7	0.067533	0.041504
		8	0.059028	0.049472
		9	0.051078	0.054942
		10	0.043501	0.058259
		11	0.036127	0.059535
		12	0.028757	0.058585
		13	0.021005	0.054436
E(LU)			0.0861974	
E(CP)			0.0167491	
E(LB)				0.0504699
14	2	1	-1.111987	-1.660295
		2	2.111987	1.660295
E(LU)			0.4209971	
E(CP)			0.3634924	
E(LB)				0.4133609
14	3	1	-0.520274	-1.119457
		2	-0.038416	-0.305215
		3	1.558690	1.424672
E(LU)			0.2261226	
E(CP)			0.1853734	
E(LB)				0.2505567
14	4	1	-0.255655	-0.849175
		2	0.016684	-0.248936
		3	0.082806	-0.082796
		4	1.156165	1.180907
E(LU)			0.1555900	
E(CP)			0.1133313	
E(LB)				0.1769729

Table of Weights (continued)

N	M	I	A(N,M,I)	C(N,M,I)
14	5	1	-0.110814	-0.686385
		2	0.050270	-0.211188
		3	0.086945	-0.078145
		4	0.105318	-0.000165
		5	0.868281	0.975883
E(LU)			0.1230332	
E(CP)			0.0767400	
E(LB)				0.1358469
14	6	1	-0.022050	-0.577438
		2	0.072538	-0.183856
		3	0.091317	-0.072778
		4	0.099530	-0.007270
		5	0.102765	0.036309
		6	0.655899	0.805033
E(LU)			0.1058306	
E(CP)			0.0556259	
E(LB)				0.1099321
14	7	1	0.036361	-0.499442
		2	0.088118	-0.163053
		3	0.095221	-0.067565
		4	0.096732	-0.011006
		5	0.095672	0.026838
		6	0.093093	0.053524
		7	0.494803	0.660706
E(LU)			0.0959410	
E(CP)			0.0424204	
E(LB)				0.0922989
14	8	1	0.076676	-0.440959
		2	0.099418	-0.146660
		3	0.098531	-0.062764
		4	0.095416	-0.012915
		5	0.091356	0.020575
		6	0.086757	0.044334
		7	0.081767	0.061525
		8	0.370080	0.536865
E(LU)			0.0899307	
E(CP)			0.0337014	
E(LB)				0.0796505
14	9	1	0.105422	-0.395636
		2	0.107819	-0.133415
		3	0.101280	-0.058429
		4	0.094871	-0.013775
		5	0.088653	0.016315
		6	0.082581	0.037749
		7	0.076590	0.053362
		8	0.070604	0.064685
		9	0.272180	0.429143
E(LU)			0.0861408	
E(CP)			0.0277259	
E(LB)				0.0702290
14	10	1	0.126370	-0.359665
		2	0.114164	-0.122519
		3	0.103539	-0.054550
		4	0.094734	-0.014009
		5	0.086936	0.013365
		6	0.079771	0.032923
		7	0.073021	0.047234
		8	0.066534	0.057697
		9	0.060185	0.065111
		10	0.194746	0.334412
E(LU)			0.0836949	
E(CP)			0.0235260	
E(LB)				0.0630171
14	11	1	0.141831	-0.330639
		2	0.118997	-0.113445
		3	0.105380	-0.051095
		4	0.094811	-0.013864
		5	0.085841	0.011310
		6	0.077858	0.029333
		7	0.070532	0.042561
		8	0.063651	0.052284
		9	0.057061	0.059247
		10	0.050632	0.063872
		11	0.133404	0.250436
E(LU)			0.0820981	
E(CP)			0.0205283	
E(LB)				0.0573895
14	12	1	0.153291	-0.306980
		2	0.122683	-0.105837
		3	0.106863	-0.048033
		4	0.094992	-0.013492
		5	0.085151	0.009885
		6	0.076555	0.026642
		7	0.068791	0.038967
		8	0.061603	0.048056
		9	0.054814	0.054608
		10	0.048286	0.059029
		11	0.041890	0.061510
		12	0.085081	0.175646
E(LU)			0.0810554	
E(CP)			0.0183757	
E(LB)				0.0529456

Table of Weights (continued)

N	M	I	A(N,M,I)	C(N,M,I)
14	13	1	0.161732	-0.287649
		2	0.125468	-0.099457
		3	0.108038	-0.045341
		4	0.095210	-0.012991
		5	0.084727	0.008915
		6	0.075676	0.024630
		7	0.067583	0.036201
		8	0.060159	0.044750
		9	0.053211	0.050936
		10	0.046592	0.055148
		11	0.040174	0.057580
		12	0.033825	0.058249
		13	0.047603	0.109028
E(LU)			0.0803840	
E(CP)			0.0168379	
E(LB)				0.0494235
14	14	1	0.167807	-0.272004
		2	0.127523	-0.094166
		3	0.108942	-0.043015
		4	0.095428	-0.012430
		5	0.084482	0.008285
		6	0.075102	0.023149
		7	0.066767	0.034099
		8	0.059168	0.042197
		9	0.052097	0.048066
		10	0.045399	0.052075
		11	0.038947	0.054419
		12	0.032614	0.055130
		13	0.026238	0.054008
		14	0.019487	0.050186
E(LU)			0.0799669	
E(CP)			0.0157637	
E(LB)				0.0466571
15	2	1	-1.201113	-1.701389
		2	2.201113	1.701389
E(LU)			0.4356936	
E(CP)			0.3748204	
E(LB)				0.4141960
15	3	1	-0.581443	-1.147269
		2	-0.067734	-0.327455
		3	1.649177	1.474724
E(LU)			0.2319583	
E(CP)			0.1926365	
E(LB)				0.2512839
15	4	1	-0.303067	-0.870352
		2	-0.005920	-0.265966
		3	0.067988	-0.098174
		4	1.240999	1.234491
E(LU)			0.1574223	
E(CP)			0.1184915	
E(LB)				0.1775276
15	5	1	-0.149966	-0.703527
		2	0.031580	-0.225104
		3	0.074752	-0.090803
		4	0.097382	-0.011644
		5	0.946252	1.031078
E(LU)			0.1226473	
E(CP)			0.0805991	
E(LB)				0.1362384
15	6	1	-0.055641	-0.591821
		2	0.056412	-0.195696
		3	0.080816	-0.083622
		4	0.092707	-0.017180
		5	0.098702	0.027353
		6	0.727003	0.860966
E(LU)			0.1040639	
E(CP)			0.0585914	
E(LB)				0.1101754
15	7	1	0.006808	-0.511778
		2	0.073806	-0.173401
		3	0.085894	-0.077113
		4	0.090667	-0.019795
		5	0.092053	0.018830
		6	0.091446	0.046348
		7	0.559325	0.716908
E(LU)			0.0932474	
E(CP)			0.0447275	
E(LB)				0.0924055
15	8	1	0.050219	-0.451675
		2	0.086466	-0.155874
		3	0.090063	-0.071341
		4	0.089895	-0.020864
		5	0.088043	0.013278
		6	0.085225	0.037735
		7	0.081744	0.055694
		8	0.428346	0.593047
E(LU)			0.0865813	
E(CP)			0.0354982	
E(LB)				0.0796276
15	9	1	0.081441	-0.404998
		2	0.095928	-0.141728
		3	0.093470	-0.066247
		4	0.089753	-0.021077
		5	0.085558	0.009563
		6	0.081118	0.031596
		7	0.076509	0.047867
		8	0.071740	0.059933
		9	0.324483	0.485091
E(LU)			0.0823091	
E(CP)			0.0291114	
E(LB)				0.0700795

Table of Weights (continued)

N	M	I	A(N,M,I)	C(N,M,I)
15	10	1	0.104439	-0.367838
		2	0.103132	-0.130087
		3	0.096250	-0.061756
		4	0.089923	-0.020801
		5	0.083997	0.007041
		6	0.078345	0.027114
		7	0.072878	0.042001
		8	0.067519	0.053113
		9	0.062195	0.061278
		10	0.241321	0.389934
E(LU)			0.0794979	
E(CP)			0.0245690	
E(LB)				0.0627397
15	11	1	0.121650	-0.337711
		2	0.108683	-0.120370
		3	0.098512	-0.057796
		4	0.090243	-0.020241
		5	0.083014	0.005320
		6	0.076446	0.023790
		7	0.070322	0.037526
		8	0.064499	0.047827
		9	0.058870	0.055459
		10	0.053342	0.060877
		11	0.174419	0.305320
E(LU)			0.0776179	
E(CP)			0.0212780	
E(LB)				0.0569788
15	12	1	0.134644	-0.312976
		2	0.112984	-0.112183
		3	0.100350	-0.054298
		4	0.090616	-0.019531
		5	0.082404	0.004159
		6	0.075137	0.021299
		7	0.068507	0.034072
		8	0.062321	0.043681
		9	0.056444	0.050840
		10	0.050769	0.055980
		11	0.045200	0.059344
		12	0.120623	0.229613
E(LU)			0.0763516	
E(CP)			0.0188676	
E(LB)				0.0523905
15	13	1	0.144470	-0.292521
		2	0.116315	-0.105250
		3	0.101830	-0.051217
		4	0.090993	-0.018746
		5	0.082037	0.003395
		6	0.074238	0.019428
		7	0.067220	0.031392
		8	0.060750	0.040410
		9	0.054673	0.047153
		10	0.048871	0.052029
		11	0.043246	0.055278
		12	0.037699	0.056995
		13	0.077657	0.161654
E(LU)			0.0755009	
E(CP)			0.0170968	
E(LB)				0.0487043
15	14	1	0.151845	-0.275596
		2	0.118870	-0.099386
		3	0.103006	-0.048518
		4	0.091344	-0.017940
		5	0.081829	0.002919
		6	0.073630	0.018031
		7	0.066315	0.029316
		8	0.059627	0.037832
		9	0.053392	0.044213
		10	0.047485	0.048847
		11	0.041802	0.051964
		12	0.036251	0.053671
		13	0.030724	0.053940
		14	0.043881	0.100709
E(LU)			0.0749378	
E(CP)			0.0158045	
E(LB)				0.0457385
15	15	1	0.157255	-0.261738
		2	0.120784	-0.094483
		3	0.103914	-0.046191
		4	0.091651	-0.017156
		5	0.081726	0.002653
		6	0.073230	0.017008
		7	0.065696	0.027731
		8	0.058844	0.035827
		9	0.052488	0.041899
		10	0.046496	0.046315
		11	0.040762	0.049298
		12	0.035191	0.050957
		13	0.029685	0.051279
		14	0.024108	0.050064
		15	0.018170	0.046538
E(LU)			0.0745778	
E(CP)			0.0148822	
E(LB)				0.0433763
16	2	1	-1.285091	-1.739679
		2	2.285091	1.739679
E(LU)			0.4499326	
E(CP)			0.3852637	
E(LB)				0.4149750

$$\begin{vmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1m} & 1 & E_1 \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2m} & 1 & E_2 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_{mm} & 1 & E_m \\ 1 & 1 & \cdots & 1 & 0 & 0 \\ E_1 & E_2 & \cdots & E_m & 0 & 0 \end{vmatrix} \begin{vmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \\ \lambda_1 \\ \lambda_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \end{vmatrix} \tag{25}$$

and

$$\begin{vmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1m} & 1 & E_1 \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2m} & 1 & E_2 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_{mm} & 1 & E_m \\ 1 & 1 & \cdots & 1 & 0 & 0 \\ E_1 & E_2 & \cdots & E_m & 0 & 0 \end{vmatrix} \begin{vmatrix} c_1 \\ c_2 \\ \cdots \\ c_m \\ \lambda_3 \\ \lambda_4 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 1 \end{vmatrix} \tag{26}$$

It can be verified that $-\lambda_1$ and $-\lambda_4$ are equal to $\mathrm{Var}(u^*_{m,n})$ and $\mathrm{Var}(b^*_{m,n})$, respectively, and that $-\lambda_2$ and $-\lambda_3$ are both equal to $\mathrm{CoV}(u^*_{m,n}, b^*_{m,n})$. If we let $\mathrm{var}(u^*_{m,n})=\alpha_{m,n}b^2$, $\mathrm{Var}(b^*_{m,n})=\gamma_{m,n}b^2$, and $\mathrm{Cov}(u^*_{m,n}, b^*_{m,n})=\beta_{m,n}b^2$ respectively, then we find that the appropriate weights $A_{i,m,n}$ and $C_{i,m,n}$ can be written as $A_{i,m,n}=a_{i,m,n}-\beta_{m,n}c_{i,m,n}/(1+\gamma_{m,n})$ and $C_{i,m,n}=c_{i,m,n}/(1+\gamma_{m,n})$, $i=1,2,\cdots,m$, respectively. The mean squred errors of $\tilde{u}_{m,n}$ and $\tilde{b}_{m,n}$ are $b^2E(LU)=[\alpha_{m,n}-\beta^2_{m,n}/(1+\gamma_{m,n})]b^2$ and $b^2E(LB)=[\gamma_{m,n}/(1+\gamma_{m,n})]b^2$, respectively. The expected value of $(\tilde{u}_{m,n}-u_{m,n})(\tilde{b}_{m,n}-b_{m,n})$ is $b^2E(CP)=[\beta_{m,n}/(1+\gamma_{m,n})]b^2$ and the mean squares error of $\tilde{X}_{P,m,n}$ is $[\alpha_{m,n}-2\beta_{m,n}\mathrm{lnln}(1/P)+\gamma_{m,n}\{\mathrm{lnln}(1/P)\}^2-\{\beta_{m,n}-\gamma_{m,n}\mathrm{lnln}(1/P)\}^2/(1+\gamma_{m,n})]b^2$. It should be noted that the mean squared errors of $\tilde{u}_{m,n}$, $\tilde{b}_{m,n}$, and $\tilde{X}_{P,m,n}$ are uniformly smaller than those of the corresponding best linear unbiased cstimators. The weights for obtaining the best linear invariant estimates of u and b and the expected losses of the estimators are given in Table 1.

4. An application to Meteorology

In ordes to illusrate the use of the **BLI** weights given in Table 1 we consider the following problem. We wish to design the probable annual maximum rainfall. Based on prior knowledge with similar situations, we are willing to assume that the annual maximum daily rainfalls have a Type 1 asymptotic distribution of the largest extreme. The problem then becomes one of estimating parameters from complete and censored sampler. For the case under consideration, the annual maximum daily rainfalls in millimeters, observed for 19 years (1959-1977) in Uchinomi, Kagawa Prefecture, are 53, 57, 66, 72, 72, 74, 88, 101, 107, 116, 132, 139, 148, 200, 216, 257, 267, 289, 758. We see that the largest obsevation is badly out of line. In the present case such a wild observation ought to be removed from the samples. We can thus use the weights with $n=19$ and $m=18$ to estimate x_P, say $x_{.99}$. Table 2 gives an example of the use of the weights. The censoring model is indended to overcome situations where one or more of the sample are spurious, giving rise to unstable parameter estimates which are unsatisfactory.

Table 2. An example of the use of the BLI weights.

ordered annual maximum daily rainfalls in mm	weights	
X_i	$A_{i,m,n}$	$C_{i,m,n}$
53.0	0.122191	−0.237435
57.0	0.098334	−0.096509
66.0	0.086816	−0.055388
72.0	0.078395	−0.030290
72.0	0.071540	−0.012826
74.0	0.065666	0.000124
88.0	0.060416	0.010117
101.0	0.055734	0.017942
107.0	0.051314	0.024310
116.0	0.047294	0.029256
132.0	0.043404	0.033344
139.0	0.039670	0.036422
148.0	0.036062	0.038827
200.0	0.032648	0.040469
215.0	0.029160	0.041489
257.0	0.025746	0.041806
267.0	0.022274	0.041382
289.0	0.033335	0.076960

$\tilde{a}=\sum_{i=1}^{18}A_iX_i=106.96536 \quad \tilde{b}=\sum_{i=1}^{18}C_iX_i=60.12029$

$\ln\ln(1/.99)=4.4001$

$\tilde{X}_{.99}=\tilde{a}-\tilde{b}\ \ln\ln(1/.99)=383.5$

$MSE(\tilde{X}_{.99})=.92104b^2$

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