Plane Algebraic Curves Drawn by a Spirograph and the Isogonal Conjugate for a Triangle

by

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Abstract

Arbitrary curves drawn by a spirograph can be obtained as loci of the isogonal conjugate to a point for an equilateral triangle which moves along a circle.

§ 1. Introduction

In the sequel to [4]-[9], [16]-[17] our study aims to develop a drawing tool on a display for experimental research on various curves using computers.

In elementary geometry we have five significant notions for a triangle; that is, the center of gravity, the center of an inscribed circle, the center of an escribed circle, the circumcenter and the orthocenter of a triangle. As a similar notion we have the isogonal conjugate to a point for a triangle.

Which curve is drawn as a locus of the isogonal conjugate to a point for a triangle which moves under a certain condition? In this study we limit ourselves to the case where a triangle is fixed while the point moves along a distinguished curve. Then our main concern is to find various curves with simple algebraic expressions as a locus of the isogonal conjugate to a point for a triangle.

Watching the display, the first author noticed that some of them were quite similar to ones drawn by a spirograph. Then we have a following question:

Can a curve drawn by a spirograph be obtained by a locus of an isogonal conjugate to a point for an equilateral triangle which moves along a simple curve?

In this paper we shall give an affirmative answer to it.

For terminology of geometry throughout the paper, consult [2], [3], [11], [15], [18] and [19].

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§ 2. A program for drawing a locus

On the plane let us consider a given triangle $\triangle ABC$ and a point $P$ different from the vertices of the triangle (Fig. 1).

Reflect the lines connecting the vertices of $\triangle ABC$ with $P$ in the bisectors of the corresponding angles of the triangle. It turns out that three lines always intersect in a single point (or are parallel, i.e., intersects in the point of infinity $\infty$), which we denote $P'$. The point $P'$ is called the isogonal conjugate of $P$ for $\triangle ABC$; the map $\varphi : \mathbb{R}^2 \setminus \{ A, B, C \} \rightarrow \mathbb{R}^2 \cup \{ \infty \}$ defined by $\varphi(P) = P'$ is called the isogonal conjugation for $\triangle ABC$.

**Proposition.** For any point $P$ different from the vertices of $\triangle ABC$, it lies on the circumcircle of $\triangle ABC$ if and only if its isogonal conjugate is the point of infinity; that is, $\varphi(P) = \infty$.

Let $A(0,1)$, $B\left(\frac{-\sqrt{3}}{2}, -\frac{1}{2}\right)$, $C\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ be the vertices of an equilateral triangle, and $P(u,v)$ a point different from the vertices. The point $P'(x, y)$ is determined as the intersection of two straight lines $P'B$ and $P'C$ given by

\[
\begin{align*}
P'B : & \quad y = m_b \left(x + \frac{\sqrt{3}}{2}\right) - \frac{1}{2}, \\
P'C : & \quad y = m_c \left(x - \frac{\sqrt{3}}{2}\right) - \frac{1}{2},
\end{align*}
\]

where

\[
\begin{align*}
m_b &= \frac{\sqrt{3} u - v + 1}{u + \sqrt{3} v + \sqrt{3}}, \\
m_c &= \frac{-\sqrt{3} u - v + 1}{u - \sqrt{3} v - \sqrt{3}}.
\end{align*}
\]

Which curve is drawn as a locus of the isogonal conjugate $P'$ to a point $P$ for $\triangle ABC$ which moves along a distinguished curve $\mathcal{C}$?
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We divide our operation of a drawing tool into the drawing part and the printing part, due to the circumstances of our computer machines.

PART ONE: To draw a locus of the isogonal conjugate to a point for a given triangle.

A program of our drawing game for Case 1 of Example 1 in § 3 is written in Visual Basic Ver. 6.0 by Microsoft Corporation and consists of the following four steps (see List 1).

**Step 1.** Set the coordinates axes and \( \triangle ABC \) in black.

**Step 2.** Set the initial position of a point \( P \) on curve \( \mathcal{C} \) in blue.

**Step 3.** Plot the isogonal conjugate \( P' \) in red.

**Step 4.** When one moves the point \( P \) continuously along the curve \( \mathcal{C} \) by the mouse, the point \( P' \) continuously draws a locus \( \mathcal{L} \) with a solid line in red.

PART TWO: To process a bitmap file (bmp) by \( \text{pLX2E}X2\epsilon \), to exhibit it on another display, and to print it.

OUTLINE of the PROGRAM. Let \( \triangle ABC \) be the equilateral triangle as above and \( \mathcal{C} : x^2 + y^2 = 4 \). Where a point \( P(u, v) \) on \( \mathcal{C} \) is selected by the mouse, then the isogonal conjugate \( P'(x, y) \) to \( P \) is determined by the equations (⋆).

The program for drawing a locus \( \mathcal{L} \) of the point \( P' \) is given in List 1. In this case we will have the curve

\[
\mathcal{L} : 3x^4 + 6x^2y^2 + 3y^4 - 6x^2y + 2y^3 - 9x^2 - 9y^2 + 4 = 0
\]

on a display (Fig. 2).

We keep the same notations as in this section throughout the paper.
As a locus of the isogonal conjugate to a point for a given equilateral triangle \( \triangle ABC \) we have various algebraic curves.

**Example 1.** Each of the following algebraic curves \( \mathcal{L} \) can be obtained as a locus of the isogonal conjugate \( P' \) to a point \( P \) for the equilateral triangle \( \triangle ABC \) which moves along the curve \( \mathcal{C} \).

**Case 1 (Fig. 2).** \( \mathcal{C} : x^2 + y^2 = 4 \), the circle.
\[
\mathcal{L} : 3x^4 + 6x^2y^2 + 3y^4 - 6x^2y + 2y^3 - 9x^2 - 9y^2 + 4 = 0.
\]

![Fig. 2](image)

**Case 2 (Fig. 3).** \( \mathcal{C} : x^2 + y^2 = \frac{1}{4} \), the circle.
\[
\mathcal{L} : 3x^4 + 6x^2y^2 + 3y^4 + 24x^2y - 8y^3 + 6x^2 + 6y^2 - 1 = 0.
\]

![Fig. 3](image)
Proof. Let $P(u, v)$ and $P'(x, y)$.

Case 1. Find the Groebner bases (see [1], [12]) by applying Mathematica Ver 6.0 by Wolfram Research Inc., to the following code:

```math
f1 := u^2 + v^2 - 4
f2 := (u + Sqrt[3]*v + Sqrt[3])*mb - (Sqrt[3]*u - v + 1)
f3 := (u - Sqrt[3]*v - Sqrt[3])*mc - (-Sqrt[3]*u - v + 1)
f4 := (2*y + 1) - mb*(2*x + Sqrt[3])
f5 := (2*y + 1) - mc*(2*x - Sqrt[3])
GroebnerBasis[{f1, f2, f3, f4, f5}, {u, v, mb, mc, x, y}]
```

Then we obtain the term

$$4 - 9x^2 + 3x^4 - 6x^2y - 9y^2 + 6x^2y^2 + 2y^3 + 3y^4$$

in the list of the generating Groebner bases. Therefore, we have

$$3x^4 + 6x^2y^2 + 3y^4 - 6x^2y + 2y^3 - 9x^2 - 9y^2 + 4 = 0$$

as an equation of $\mathcal{L}$.

The proof for case 2 is omitted. □
§ 3. Results

A spirograph is a geometric drawing toy that produces various curves known as hypotrochoids and epitrochoids.

Let the fixed circle have radius $a$ and have its center at the origin $O$ of an $xy$-coordinate system (Fig. 4). Let the rolling circle have radius $r (r > 0)$ and have center $P$ which travels with it, and let $\theta$ be the angle which $\overrightarrow{OP}$ makes with the $y$-axis so that

$$\overrightarrow{OP} = (a - r)(-\sin \theta, \cos \theta).$$

We suppose that $\theta > 0$, so the spoke $PR$ in the moving wheel rotates in a positive direction, while the wheel (the circle) $P$ contacts with the circle $O$ at $S$, and $Q$ the point on the spoke (or the extended spoke) $PR$. For $\theta = 0$, denote the initial position of the points $P, Q, R$ by $P_0, Q_0, R_0$, respectively. Let $Q_0 = (0, q_0)$ and $t = a - q_0 \neq r$. 

![Fig. 4]

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We have the following:

**Theorem 1.** *Parametric equations for the locus \( \mathcal{K} \) of the point \( Q \) are given by*

\[
x = -(a-r) \sin \theta - (r-t) \sin \left( \frac{r-a}{r} \right) \theta ,
\]
\[
y = (a-r) \cos \theta + (r-t) \cos \left( \frac{r-a}{r} \right) \theta .
\]

*Proof.* Let \( \alpha = \angle RPS \). Since \( \overrightarrow{SR} = \overrightarrow{SR}_0 \), we have

\[
r \alpha = a \theta , \quad \text{or} \quad \alpha = \frac{a}{r} \theta .
\]

Since the angle which \( \overrightarrow{PR} \) makes with the \( x \)-axis equals \( \frac{\pi}{2} + \theta - \alpha \), we have

\[
\overrightarrow{PQ} = (r-t) \left( - \sin \left( \frac{r-a}{r} \right) \theta , \cos \left( \frac{r-a}{r} \right) \theta \right).
\]

Therefore, we have the conclusion because \( \overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} \). \( \Box \)
Example 2. Each of the following algebraic curves $\mathcal{H}$ can be obtained as a locus of the point $Q$ in Theorem 1.

Case 1 (Fig. 5). $a = 1, \; r = \frac{1}{3}, \; t = \frac{5}{3}$.

$$\mathcal{H} : 3x^4 + 6x^2y^2 + 3y^4 - 6x^2y + 2y^3 - 9x^2 - 9y^2 + 4 = 0.$$ 

![Figure 5](image)

Case 2 (Fig. 6). $a = 1, \; r = \frac{1}{3}, \; t = 0$.

$$\mathcal{H} : 3x^4 + 6x^2y^2 + 3y^4 + 24x^2y - 8y^3 + 6x^2 + 6y^2 - 1 = 0.$$ 

![Figure 6](image)
Proof.

**Case 1.** In this case, we have

\[
x = \frac{1}{3} (-2 \sin \theta - 4 \sin 2\theta),
\]

\[
y = \frac{1}{3} (2 \cos \theta - 4 \cos 2\theta).
\]

Find the Groebner bases by applying Mathematica to the following code:

\[
f1 := c^2 + s^2 - 1
\]
\[
c2 := 2*c^2 - 1
\]
\[
s2 := 2*c*s
\]
\[
f2 := x - (-2*s - 4*s2)/3
\]
\[
f3 := y - (2*c - 4*c2)/3
\]
\[
\text{GroebnerBasis}\{\{f1, f2, f3\}, \{s, c, y, x\}\}
\]

Then we obtain the term

\[4 - 9x^2 + 3x^4 - 6x^2y - 9y^2 + 6x^2y^2 + 2y^3 + 3y^4\]

in the list of the generating Groebner bases. Therefore, we have

\[3x^4 + 6x^2y^2 + 3y^4 - 6x^2y + 2y^3 - 9x^2 - 9y^2 + 4 = 0\]

as an equation of \(X\).

The proof for Case 2 is omitted. \(\square\)
Theorem 2. The algebraic curve

\[ L : (b^2 - 1)x^4 + 2(b^2 - 1)x^2y^2 + (b^2 - 1)y^4 - 6x^3y + 2y^3 - (2b^2 + 1)x^2 - (2b^2 + 1)y^2 + b^2 = 0 \]

can be obtained as a locus of the isogonal conjugate \( P' \) to a point \( P \) for the equilateral triangle \( \triangle ABC \) which moves along the circle

\[ C : x^2 + y^2 = b^2 \quad \quad (b > 0, \ b \neq 1). \]

Proof. Let \( P(u, v) \) and \( P'(x, y) \). Find the Groebner bases by applying Mathematica to the following code:

\[
\begin{align*}
\text{f1} & := u^2 + v^2 - b^2 \\
\text{f2} & := (u + \text{Sqrt}[3]*v + \text{Sqrt}[3])*mb - (\text{Sqrt}[3]*u - v + 1) \\
\text{f3} & := (u - \text{Sqrt}[3]*v - \text{Sqrt}[3])*mc - (-\text{Sqrt}[3]*u - v + 1) \\
\text{f4} & := (2y + 1) - mb*(2x + \text{Sqrt}[3]) \\
\text{f5} & := (2y + 1) - mc*(2x - \text{Sqrt}[3]) \\
\text{GroebnerBasis} & \{\text{f1, f2, f3, f4, f5}, \{u, v, mb, mc, x, y\}\}
\end{align*}
\]

Then we obtain the term

\[-b^2 + (1 + 2b^2)x^2 + (1 - b^2)x^4 + 6x^3y + (1 + 2b^2)y^2 + (2 - 2b^2)x^2y^2 - 2y^3 + (1 - b^2)y^4\]

in the list of the generating Groebner bases. Therefore, we have

\[(b^2 - 1)x^4 + 2(b^2 - 1)x^2y^2 + (b^2 - 1)y^4 - 6x^3y + 2y^3 - (2b^2 + 1)x^2 - (2b^2 + 1)y^2 + b^2 = 0\]

as an equation of \( L \). □
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Theorem 3. Let \( f(x, y) \), \( g(x, y) \) be polynomials that define the curves \( \mathcal{H} \) in Theorem 1, \( \mathcal{L} \) in Theorem 2, respectively; that is,

\[
\begin{align*}
    f(x, y) &= -(r-t)^2x^4 - 2(r-t)^2x^2y^2 - (r-t)^2y^4 - 24r^2(r-t)x^2y \\
    &\quad + 8r^2(r-t)y^3 - 2(3r-t)(r+t)(3r^2 - 2rt + t^2)x^2 \\
    &\quad - 2(3r-t)(r+t)(3r^2 - 2rt + t^2)y^2 + (3r-t)^3(r+t)^3, \\
    g(x, y) &= (b^2 - 1)x^4 + 2(b^2 - 1)x^2y^2 + (b^2 - 1)y^4 - 6x^2y + 2y^3 \\
    &\quad - (2b^2 + 1)x^2 - (2b^2 + 1)y^2 + b^2, \\
\end{align*}
\]

In both curves \( f(x, y) = 0 \) and \( g(x, y) = 0 \) coincide, then it holds that

\[
r = \frac{b}{2(b^2 - 1)}, \quad t = \frac{b(2b \pm 1)}{2(b^2 - 1)}. 
\]

Proof. Put \( f(x, y) = kg(x, y) \). By comparing the coefficients of both sides we have

\[
\begin{align*}
    - (r-t)^2 &= k(b^2 - 1) \\
    (3r-t)^3(r+t)^3 &= kb^2 \\
    8r^2(r-t) &= 2k \\
    - 2(3r-t)(r+t)(3r^2 - 2rt + t^2) &= k(-2b^2 - 1). 
\end{align*}
\]

Find the Gröbner bases by applying Mathematica to the following code:

\[
\begin{align*}
    f1 &= -(r - t)^2 - k*(b^2 - 1) \\
    f2 &= (3*r - t)^3*r + t) - 3 - k*b^2 \\
    f3 &= 8*r^2*(r - t) - k*2 \\
    f4 &= -2*(3*r - t)*(r + t)*(3*r^2 - 2*r*t + t^2) - k*(-2*b^2 - 1) \\
    GroebnerBasis[\{f1, f2, f3, f4\}, \{t, k, b, r\}] \\
    Factor[%] \\
\end{align*}
\]

Then we obtain the term

\[
r^4(-1 + 3r)^2(1 + 3r)^2(-b - 2r + 2b^2r)(b - 2r + 2b^2r)
\]

in the list of the generating Gröbner bases. By \( r > 0 \), we have

\[
(1 + 3r)^2(-b - 2r + 2b^2r)(b - 2r + 2b^2r) = 0. \quad (1)
\]
On the other hand, find the Groebner bases by applying Mathematica to the following code:

\[
\begin{align*}
  f_1 &:= -(r - t)^2 - k^* (b^2 - 1) \\
  f_2 &:= (3r - t)^3 (r + t)^3 - k^* b^2 \\
  f_3 &:= 8r^2 (r - t) - k^* b^2 \\
  f_4 &:= -2(3r - t)(r + t)(3r^2 - 2r + t - 2) - k^*(2b^2 - 1) \\
  \text{GroebnerBasis} &\left[\{f_1, f_2, f_3, f_4\}, \{r, k, b, t\}\right] \\
  \text{Factor} &\left[\%\right]
\end{align*}
\]

Then we obtain the term

\[
t^4 (2 + 3t)^2 (b - 2b^2 - 2t + 2b^2 t) (b - 2b^2 - 2t + 2b^2 t)
\]

in the list of the generating Groebner bases.
Hence, we have

\[
t^4 (2 + 3t)^2 (b - 2b^2 - 2t + 2b^2 t) (b - 2b^2 - 2t + 2b^2 t) = 0. \tag{2}
\]

By (1), (2), we have the following:
If \(-1 + 3r \neq 0\) and \(t(2 + 3t) \neq 0\), then

\[
r = \left| \frac{b}{2(b^2 - 1)} \right|, \quad t = \frac{b(2b \pm 1)}{2(b^2 - 1)}.
\]

The results also hold for the case \(-1 + 3r = 0\) or \(t(2 + 3t) = 0\). □
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As an application to Theorem 3 we have the following:

**Example 3 (Fig. 7).** The algebraic curve

\[ \mathcal{L} : 7x^4 + 14x^2y^2 + 7y^4 - 32y^3 + 34x^2 + 34y^2 - 9 = 0 \]

can be obtained as a locus of the isogonal conjugate \( P' \) to a point \( P \) for the equilateral triangle \( \triangle ABC \) which moves along the circle

\[ \mathcal{C} : x^2 + y^2 = \frac{9}{16}. \]

![Fig. 7](image)

**Example 4 (Fig. 8).** For \( b = \frac{3}{4}, \ r = \frac{6}{7}, \ t = -\frac{3}{7} \), the locus of the point \( Q \) in Theorem 1 is given by

\[ \mathcal{K} : 7x^4 + 14x^2y^2 + 7y^4 - 32y^3 + 34x^2 + 34y^2 - 9 = 0, \]

which coincides to the curve \( \mathcal{L} \) in Example 3.

![Fig. 8](image)
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References


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List 1. A program for drawing a locus

' Specifying statements of the general variables
Dim sx, sy, ex, ey As Double
Dim ax, ay, bx, by, cx, cy As Double
Dim pi As Double
Dim tb, tc As Double
Dim k As Integer
Dim j As Double
Dim qr As Double

' Drawing the initial figure
Private Sub Form_Activate()
Dim r, X, Y As Double

Form1.AutoRedraw = True
Form1.Line (sx, 0)-(ex, 0)
Form1.Line (0, sy)-(0, ey)

sxn = Int(sx): syn = Int(sy)
exn = Int(ex): eyn = Int(ey)

h = 0.04
For i = sxn To exn
   Form1.Line (i, -h)-(i, h)
Next i

For i = syn To eyn
   Form1.Line (-h, i)-(h, i)
Next i

Form1.DrawWidth = 2
Form1.Line (ax, ay)-(bx, by), vbBlack
Form1.Line (cx, cy)-(ax, ay), vbBlack
Form1.Line (bx, by)-(cx, cy), vbBlack

Form1.AutoRedraw = False
Form1.Circle (0, 0), 1.5, vbBlue

tb = AGL(ax, ay, bx, by, bx + 1, by)
tc = AGL(ax, ay, cx, cy, cx + 1, cy)

k = 1
Form1.DrawWidth = 4
j = 0
Equ_Agl 1.5
qr = 1.5
Text1.Text = qr
End Sub
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Private Sub Form_Load()
    Form1.Top = 50
    Form1.Left = 500
    Form1.Height = 0.95 * Screen.Height
    Form1.Width = 0.7 * Screen.Width
    sx = -4.2
    ex = 4.2
    wx = ex - sx
    wy = wx * Form1.ScaleHeight / Form1.ScaleWidth
    sy = -0.5 * wy
    ey = 0.5 * wy
    Form1.Scale (sx, ey)-(ex, sy)
    Form1.BackColor = vbWhite
    Form1.DrawWidth = 1
    pi = 4 * Atn(1)
    ax = 0: ay = 1
    bx = -Sqr(3) / 2: by = -0.5
    cx = Sqr(3) / 2: cy = -0.5

    Text1.FontSize = 16
End Sub

Private Sub Form_MouseDown(Button As Integer, Shift As Integer, X As Single, Y As Single)
    If k = 2 Then
        k = 1
        Exit Sub
    End If

    Form1.Cls
    Form1.DrawWidth = 2
    r = Sqr(X ^ 2 + Y ^ 2)
    Form1.Circle (0, 0), r, vbBlue
    Form1.DrawWidth = 4

    Equ_Agl r
    qr = r
    Text1.Text = qr
    k = 2
End Sub
Private Sub Form_MouseMove(Button As Integer, Shift As Integer, X As Single, Y As Single)
If k = 1 Then Exit Sub

Form1.Cls
r = Sqr(X ^ 2 + Y ^ 2)
Form1.Circle (0, 0), r, vbBlue
Form1.DrawWidth = 4

Equ_Angle r
qr = r
Text1.Text = qr
End Sub

Private Function Angle(xa, ya, xb, yb, xc, yc)
a = Sqr((xc - xb) ^ 2 + (yc - yb) ^ 2)
b = Sqr((xa - xc) ^ 2 + (ya - yc) ^ 2)
c = Sqr((xb - xa) ^ 2 + (yb - ya) ^ 2)
If a < 0.0000001 Or c < 0.0000001 Then Exit Function
X = (a ^ 2 + c ^ 2 - b ^ 2) / (2 * a * c)
If 1 - X ^ 2 < 0.0000001 Then
  t = 0
Else
  t = pi
End If
Else
  t = pi / 2 - Atn(X / Sqr(1 - X ^ 2))
End If
h = xa * yb + xb * yc + xc * ya - xa * yc - xb * ya - xc * yb
If h < 0 Then t = -t
Angle = t
End Function
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' Presenting of the isogonal conjugate

Private Sub Disp(px, py)
    sb = Agl(px, py, bx, by, cx, cy)
    mb = Tan(tb - sb)
    sc = Agl(px, py, cx, cy, bx, by)
    mc = Tan(tc - sc)
    qx = (mb * bx - mc * cx - by + cy) / (mb - mc)
    qy = mb * (qx - bx) + by
    If Abs(qx) > ex Or Abs(qy) > ey Then Exit Sub
    Form1.PSet (qx, qy), Q8Color(12)
End Sub

Private Sub Equ_Agl(r)
    For t = 0 To 2 * pi Step 0.01
        X = r * Cos(t)
        Y = r * Sin(t)
        Disp X, Y
    Next t
End Sub

Private Sub Ext_Click()
    For t = 0 To 2 * pi Step 0.0001
        X = qr * Cos(t)
        Y = qr * Sin(t)
        Disp X, Y
    Next t
End Sub

Private Sub Text1_DblClick()
    r = Val(Text1.Text)
    Form1.Clrs
    Form1.DrawWidth = 2
    Form1.Circle (0, 0), r, vbBlue
    Form1.DrawWidth = 4
    Equ_Agl r
    qr = r
    k = 1
End Sub