A Locus of the Orthocenter of a Pedal Triangle

— Instruction of Geometry by Use of a Drawing Game on a Display —

by

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Abstract

An effective use of a computer makes mathematics classes much more interesting and motivates many students to learn. In this paper we present a computer game for drawing a locus of the orthocenter of the pedal triangle of a given triangle with two vertices fixed when one moves the third vertex along a distinguished curve. The drawing game provides an active teaching material of elementary geometry for students.

§ 1. Introduction

In the sequel to [4] - [7] our study aims to develop a drawing game on a display as a teaching material for elementary geometry classes with activity using computers.

In elementary geometry we have five significant notions for a triangle; that is, the center of gravity, the center of an inscribed circle, the center of an escribed circle, the circumcenter and the orthocenter of a triangle.

Which curve is drawn as a locus of such a point of the pedal triangle of a triangle with two vertices fixed on the plane when the third vertex moves under a certain condition?

In this study we limit ourselves to the case of a locus of the orthocenter of a pedal triangle of a triangle with two vertices fixed on the plane when the third vertex
moves along a distinguished curve. Then our main concern is to find various types of remarkable curves with a simple expression as a locus of the orthocenter of a pedal triangle.

In this paper we present a computer game for students to find a solution to the above problem with fun, and obtain some results from our various experiments. In fact, “Mathematical Discovery” due to G. Polya [11] indicates the spirit of our research in mathematics education.

For terminology of geometry throughout the paper, consult [2], [3] and [10].

As for the present paper, the first author gave a general idea for drawing a locus of the orthocenter of a pedal triangle of a given triangle on a display and wrote the programs for all of the figures under the direction of the second author. The last author joined the discussions and arranged the results for publishing.

§ 2. A program for drawing a locus

Let us consider a triangle \( \triangle ABC \) given on a plane.

Let \( A = (x_a, y_a), \quad B = (x_b, y_b), \quad C = (x_c, y_c) \). Then the coordinates of the orthocenter \( H \) of \( \triangle ABC \) is given by

\[
\left( \frac{r_1 q_2 - r_2 q_1}{p_1 q_2 - p_2 q_1}, \quad \frac{p_1 r_2 - p_2 r_1}{p_1 q_2 - p_2 q_1} \right)
\]

where

\[
p_1 = x_b - x_c, \quad q_1 = y_b - y_c, \quad r_1 = x_a (x_b - x_c) + y_a (y_b - y_c),
\]
\[
p_2 = x_a - x_c, \quad q_2 = y_a - y_c, \quad r_2 = x_b (x_a - x_c) + y_b (y_a - y_c).
\]

Let \( D, E, F \) be the feet of the perpendiculars from the vertices \( A, B, C \) to the opposite sides, respectively. The \( \triangle DEF \) is called the pedal triangle of \( \triangle ABC \). Denote the orthocenter of \( \triangle DEF \) by \( K \).

Which curve is drawn as a locus of the orthocenter \( K \) of the pedal triangle \( \triangle DEF \) of \( \triangle ABC \) with the vertices \( B, C \) fixed when the third vertex \( A \) moves along a distinguished curve \( \mathcal{C} \)?

We divide our operation of a drawing game into the drawing part and the printing part, due to the circumstances of our computer machines.

PART ONE: To draw a locus of the orthocenter of a pedal triangle of a given triangle. A program of our drawing game for Example 4 in § 3 is written in Visual Basic
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Ver. 6.0 by Microsoft Corporation and consists of the following seven steps (see List 1).

**Step 1.** Set the coordinates axes and two fixed point $B$, $C$ in black and a curve $\mathcal{C}$ in blue.

**Step 2.** Set the initial position of a vertex $A$ of a triangle $\triangle ABC$ and draw each side in black.

**Step 3.** Plot the orthocenter $H$ of $\triangle ABC$ in red.

**Step 4.** Draw each of the perpendiculars from the vertices of $\triangle ABC$ to the opposite sides with a broken line in blue.

**Step 5.** Draw the sides of the pedal triangle $\triangle DEF$ of $\triangle ABC$ in magenta and plot its orthocenter $K$ in indigo-blue.

**Step 6.** When $\triangle ABC$ or $\triangle DEF$ is obtuse, extend each of two sides adjacent at the obtuse angle with a broken line in green.

**Step 7.** When one moves the vertex $A$ continuously along the curve $\mathcal{C}$ by the mouse, the orthocenter $K$ of $\triangle DEF$ continuously draws a locus $\mathcal{L}$ with a solid line in indigo-blue.

**PART TWO:** To process a bitmap file (bmp) by \LaTeX\texttt{2e}, to exhibit it on another display, and to print it.

**OUTLINE of the PROGRAM.** Let $B(-1, 0), C(1, 0)$ be the fixed vertices of a triangle on the plane and let $\mathcal{C}$ be the circle given by the equation $x^2 + (y - 1)^2 = 5$. When a point $A(u, v)$ on $\mathcal{C}$ is selected by the mouse, the orthocenter $K(x, y)$ is determined by the formula (*) with $D = (x_a, y_a), E = (x_b, y_b), F = (x_c, y_c)$.

The program for drawing a locus $\mathcal{L}$ of the point $K$ is given in List 1. In this case we will have the ellipse $\mathcal{L} : x^2 + \frac{y^2}{\left(\frac{3\sqrt{5}}{5}\right)^2} = 1$ on a display (Fig. 4).

We keep the same notation for the rest of the paper.
§ 3. Curves obtained as a locus of the orthocenter of a pedal triangle

As a locus of the orthocenter of a pedal triangle of a given triangle we have various known curves. In this section we exhibit the cases of circles and ellipses.

**Proposition 1.** A circle can be obtained as a locus of the orthocenter of the pedal triangle of $\triangle ABC$ when the third vertex $A$ moves along a straight line.

**Example 1.** (Fig. 1) Let $B(-1, 0)$, $C(1, 0)$ and $C: y = x + 1$. Then we have the circle $\mathcal{L}: x^2 + y^2 = 1$ as a locus of the orthocenter of the pedal triangle $\triangle DEF$ of $\triangle ABC$ when the third vertex $A$ moves along $C$.

**Proof.** Let $A(u, v)$ and $K(x, y)$, while $B = (-1, 0) = (-\alpha, -\beta), C = (1, 0) = (\alpha, -\beta)$ are fixed. Then we have the following:

$$AC: y + \beta = \frac{v + \beta}{u - \alpha} (x - \alpha),$$

and

$$BE: y + \beta = \frac{v + \alpha}{u - \beta} (x + \alpha),$$

Hence

$$E = \begin{pmatrix} \frac{-(u - 1)^2 + v^2}{(u - 1)^2 + v^2} & \frac{-2v(u - 1)}{(u - 1)^2 + v^2} \\ \frac{(u + 1)^2 - v^2}{(u + 1)^2 + v^2} & \frac{2v(u + 1)}{(u + 1)^2 + v^2} \end{pmatrix},$$

$$F = \begin{pmatrix} \frac{-(u - 1)^2 + v^2}{(u - 1)^2 + v^2} & \frac{-2v(u - 1)}{(u - 1)^2 + v^2} \\ \frac{(u + 1)^2 - v^2}{(u + 1)^2 + v^2} & \frac{2v(u + 1)}{(u + 1)^2 + v^2} \end{pmatrix},$$

since $\alpha = 1, \beta = 0$.

Find the Groebner bases (see [1], [9]) by applying Mathematica by Wolfram Research, Inc., to the following code, where $D = (x_a, y_a), E = (x_b, y_b), F = (x_c, y_c)$:
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\[
f_0 := v - u - 1 \\
f_1 := x - a - u \\
f_2 := y - a \\
f_3 := x_b - (v^2 - (u - 1)^2)/(u - 1)^2 + v^2 \\
f_4 := y_b + 2*(u - 1)*v/(u - 1)^2 + v^2 \\
f_5 := x_c - ((u + 1)^2 - v^2)/(u + 1)^2 + v^2 \\
f_6 := y_c - 2*(u + 1)*v/(u + 1)^2 + v^2 \\
f_7 := p_1 - (x_c - x_b) \\
f_8 := q_1 - (y_c - y_b) \\
f_9 := r_1 - ((x_c - x_b) + x_a + (y_c - y_b)*y_a) \\
f_{10} := p_2 - (x_a - x_c) \\
f_{11} := q_2 - (y_a - y_c) \\
f_{12} := r_2 - ((x_a - x_c) + x_b + (y_a - y_c)*y_b) \\
f_{13} := (p_1*q_2 - p_2*q_1)*x - (r_1*q_2 - r_2*q_1) \\
f_{14} := (p_1*q_2 - p_2*q_1)*y - (p_1*r_2 - p_2*r_1)
\]

\[\text{GroebnerBasis}[[f_0, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}, f_{13}, f_{14}], \\
\{x_a, y_a, x_b, y_b, x_c, y_c, p_1, p_2, q_1, q_2, r_1, r_2, v, u, y, x]\] \]

\[\text{Factor}[]\]

Then we obtain the term

\[(-1 + u)u(-1 + x^2 + y^2)\]

in the list of the generating Groebner bases.

Therefore, we have \(-1 + x^2 + y^2 = 0\), or \(x^2 + y^2 = 1\). \(\square\)

**Proposition 2.** A certain known curve can be obtained as a locus of the orthocenter of the pedal triangle of \(\triangle ABC\) when the third vertex \(A\) moves along a straight line.

**Example 2.** (Fig. 2) Let \(B(-1, 0)\), \(C(1, 0)\) and \(\mathcal{C}: x = 2\). Then we have the curve \(\mathcal{L}: y^2 = \frac{-(x-1)(x-2)^2}{x}\) as a locus of the orthocenter of the pedal triangle of \(\triangle ABC\) when the third vertex \(A\) moves along \(\mathcal{C}\).

The curve \(\mathcal{L}\) is given as Example 43 in [12, P. 65; P. 67 for its graph].

**Proposition 3.** An ellipse can be obtained as a locus of the orthocenter of the pedal triangle of \(\triangle ABC\) when the third vertex \(A\) moves along a circle.
Example 3. (Fig. 3) Let $B(-\sqrt{0.75}, -0.5), C(\sqrt{0.75}, 0.5)$ and $C: x^2 + y^2 = 1$.

Then we have the ellipse $L: \frac{x^2}{\left(\frac{3}{2}\right)^2} + \frac{(y + \frac{1}{2})^2}{\left(\frac{1}{2}\right)^2} = 1$ as a locus of the orthocenter of the pedal triangle of $\triangle ABC$ when the vertex $A$ moves along $C$.

Proposition 4. Let $B(-1, 0), C(1, 0)$ and $C: x^2 + (y-b)^2 = 1 + b^2$ a circle where $b$ is a real constant.

Then we have the ellipse $L: \frac{x^2}{\left(\frac{2}{\sqrt{b^2 + 1}}\right)^2} + \frac{y^2}{\left(\frac{(b+1)(b-1)}{\sqrt{b^2 + 1}}\right)^2} = 1$

as a locus of the orthocenter of the pedal triangle of $\triangle ABC$ when the third vertex $A$ moves along $C$.

Proof. Under the same notation as in Proof of Example 1, find the Groebner bases (see [1], [9]) by applying Mathematica to the following code:

```mathematica
f0 := u^2 + (v-b)^2 - (1+b^2)
f1 := xa - u
f2 := ya
f3 := xb - (v^2 - (u-1)^2)/((u-1)^2 + v^2)
f4 := yb + 2*(u-1)*v/((u-1)^2 + v^2)
f5 := xc - ((u+1)^2 - v^2)/((u+1)^2 + v^2)
f6 := yc - 2*(u+1)*v/((u+1)^2 + v^2)
f7 := p1 - (xc - xb)
f8 := q1 - (yc - yb)
f9 := r1 - ((xc - xb)*xa + (yc - yb)*ya)
f10 := p2 - (xa - xc)
f11 := q2 - (ya - yc)
f12 := r2 - ((xa - xc)*xb + (ya - yc)*yb)
f13 := (p1*q2 - p2*q1)*x - (r1*q2 - r2*q1)
f14 := (p1*q2 - p2*q1)*y - (p1*r2 - p2*r1)
GroebnerBasis[{f0, f1, f2, f3, f4, f5, f6, f7, f8, f9, f10, f11, f12, f13, f14},
{x, y, x, y, x, y, p1, p2, q1, q2, r1, r2, v, u, y, x}]
Factor[]
```
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Then we obtain the term

\[ b(-1+u)(1+u)(-4+8b^2-4b^4+x^2-b^2x^2-b^4x^2+b^6x^2+4y^2+4b^2y^2) \]

in the list of the generating Groebner bases. Therefore, we have

\[ (b^8-b^4-b^2+1)x^2+4(b^2+1)y^2 = 4b^4-8b^2+4; \]

that is,

\[ \frac{x^2}{\left(\frac{2}{\sqrt{b^2+1}}\right)^2} + \frac{y^2}{\left(\frac{(b+1)(b-1)}{\sqrt{b^2+1}}\right)^2} = 1. \]

\[ \boxed{} \]

**Example 4. (Fig. 4)** For \( b = 2 \) in Proposition 4 we have the ellipse \( \mathcal{L} \):

\[ x^2 + \frac{y^2}{\left(\frac{3\sqrt{5}}{5}\right)^2} = 1 \]

as a locus of the orthocenter of the pedal triangle of \( \triangle ABC \) when the vertex \( A \) moves along \( \mathcal{E} \).

\[ \boxed{} \]

§ 4. Questions

**Question 1.** Can all conics be obtained as a locus of the orthocenter of the pedal triangle of a triangle with two vertices fixed when the third vertex moves along a distinguished curve with a simple expression?

**Question 2.** Can various known curves such as listed in a text \([8; \text{pp. 520-521}]\) be obtained as a locus of the orthocenter of the pedal triangle of a triangle with two vertices fixed when the third vertex moves along a distinguished curve with a simple expression?
References


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**Fig. 1**  \( C : \ y = x + 1 \)
\( L : \ x^2 + y^2 = 1 \)

**Fig. 2**  \( C : \ x = 2 \)
\( L : \ y^2 = -\frac{(x-1)(x-2)^2}{x} \)

**Fig. 3**  \( C : \ x^2 + y^2 = 1 \)
\[ L : \frac{x^2}{\left(\frac{3}{2}\right)^2} + \frac{(y + \frac{1}{2})^2}{\left(\frac{1}{2}\right)^2} = 1 \]

**Fig. 4**  \( C : \ x^2 + (y-2)^2 = 5 \)
\[ L : \frac{x^2}{\left(\frac{3\sqrt{5}}{5}\right)^2} + \frac{y^2}{\left(\frac{3\sqrt{5}}{5}\right)^2} = 1 \]
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List 1. A program for drawing a locus

'-------------------------------------
' Specifying statements of the general variables
'-------------------------------------
Dim sx As Double, ex As Double
Dim sy As Double, ey As Double
Dim ax As Double, ay As Double
Dim bx As Double, by As Double
Dim cx As Double, cy As Double
Dim xd As Double, yd As Double
Dim xe As Double, ye As Double
Dim xf As Double, yf As Double
Dim k As Integer
Dim ra(4) As Double

'-------------------------------------
' Drawing the initial figure
'-------------------------------------
Private Sub Form_Activate()
    Form1.AutoRedraw = True
    Form1.Line (sx, 0)-(ex, 0)
    Form1.Line (0, sy)-(0, ey)
    Form1.Circle (0, 2), Sqr(5), vbBlue
    Form1.Line (bx, by)-(cx, cy)
    Form1.DrawMode = vbNotXorPen
    Form1.Line (ax, ay)-(bx, by)
    Form1.Line (cx, cy)-(ax, ay)
    Orthocenter ax, ay, bx, by, cx, cy, 12, 2, 1
    Pedal_triangle ax, ay, bx, by, cx, cy, 13
End Sub

'-------------------------------------
' Setting the form, the coordinate axes and the initial
' values of the coordinates of each vertex of a triangle
'-------------------------------------
Private Sub Form_Load()
    sx = -4.2
    ex = 4.2
    wx = ex - sx
    wy = wx * Form1.ScaleHeight / Form1.ScaleWidth
    sy = -0.3 * wy
    ey = 0.7 * wy
    Form1.Scale (sx, ey)-(ex, sy)
    Form1.BackColor = vbWhite
    Form1.DrawWidth = 1
    ax = 0: ay = 2 + Sqr(5)
    bx = -1
    by = 0
    cx = 1
    cy = 0
    k = 1
End Sub
Action corresponding to the left button of mouse

Private Sub Form_MouseDown(Button As Integer, Shift As Integer, X As Single, Y As Single)
  If k = 2 Then
    k = 1
    Exit Sub
  End If
  k = 2
  d1 = Sqr((ax - X) ^ 2 + (ay - Y) ^ 2)
  If d1 < 0.5 Then
    Form1.Line (ax, ay)-(bx, by)
    Form1.Line (cx, cy)-(ax, ay)
    r = Sqr(X ^ 2 + (Y - 2) ^ 2)
    X = Sqr(5) * X / r: Y = Sqr(5) * (Y - 2) / r + 2
    Form1.Line (X, Y)-(bx, by)
    Form1.Line (cx, cy)-(X, Y)
    Orthocenter ax, ay, bx, by, cx, cy, 12, 2, 1
    Orthocenter X, Y, bx, by, cx, cy, 12, 2, 1
    Pedal_triangle ax, ay, bx, by, cx, cy, 13
    Pedal_triangle X, Y, bx, by, cx, cy, 13
    ax = X: ay = Y
  End If
End Sub

Action corresponding to the movement of mouse

Private Sub Form_MouseMove(Button As Integer, Shift As Integer, X As Single, Y As Single)
  If k = 1 Then Exit Sub
  d1 = Sqr((ax - X) ^ 2 + (ay - Y) ^ 2)
  If d1 < 0.5 Then
    Form1.Line (ax, ay)-(bx, by)
    Form1.Line (cx, cy)-(ax, ay)
    r = Sqr(X ^ 2 + (Y - 2) ^ 2)
    X = Sqr(5) * X / r: Y = Sqr(5) * (Y - 2) / r + 2
    Form1.Line (X, Y)-(bx, by)
    Form1.Line (cx, cy)-(X, Y)
    Orthocenter ax, ay, bx, by, cx, cy, 12, 2, 1
    Orthocenter X, Y, bx, by, cx, cy, 12, 2, 1
    Pedal_triangle ax, ay, bx, by, cx, cy, 13
    Pedal_triangle X, Y, bx, by, cx, cy, 13
    ax = X: ay = Y
  End If
End Sub

Presenting of the orthocenter of a triangle

Private Sub Orthocenter(jxa, jya, jxb, jyb, jxc, jyc, pcl, ecl, beaf)
  xa = jxa: ya = jya

---
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\[ \begin{align*}
x_b &= jxb: y_b &= jyb \\
x_c &= jxc: y_c &= jyc \\
x_u &= x_b - x_a: u_y &= y_b - y_a \\
x_v &= x_a - x_c: v_y &= y_a - y_c \\
x_w &= x_c - x_b: w_y &= y_c - y_b \\
a &= \text{Sqr}(w_x \wedge 2 + w_y \wedge 2) \\
b &= \text{Sqr}(v_x \wedge 2 + v_y \wedge 2) \\
c &= \text{Sqr}(u_x \wedge 2 + u_y \wedge 2) \\
ah &= b \wedge 2 + c \wedge 2 - a \wedge 2 \\
bh &= c \wedge 2 + a \wedge 2 - b \wedge 2 \\
ch &= a \wedge 2 + b \wedge 2 - c \wedge 2 \\
\text{If } ah \geq 0 \text{ And bh} \geq 0 \text{ And ch} \geq 0 \text{ Then} \\
p_1 &= x_b \times y_c - x_c \times y_b: q_1 &= -w_y \times y_a - w_x \times x_a \\
p_2 &= x_c \times y_a - x_a \times y_c: q_2 &= -v_y \times y_b - v_x \times x_b \\
p_3 &= x_a \times y_b - x_b \times y_a: q_3 &= -u_y \times y_c - u_x \times x_c \\
x_d &= (p_1 \times w_y - q_1 \times w_x) / a \wedge 2 \\
y_d &= (-p_1 \times w_x - q_1 \times w_y) / a \wedge 2 \\
x_e &= (p_2 \times v_x - q_2 \times v_x) / b \wedge 2 \\
y_e &= (-p_2 \times v_x - q_2 \times v_y) / b \wedge 2 \\
x_f &= (p_3 \times u_x - q_3 \times u_x) / c \wedge 2 \\
y_f &= (-p_3 \times u_x - q_3 \times u_y) / c \wedge 2 \\
x_h &= (q_2 \times w_y - q_1 \times v_y) / (w_x \times v_y - w_y \times v_x) \\
y_h &= (q_1 \times v_x - q_2 \times w_x) / (w_x \times v_y - w_y \times v_x) \\
\text{Form1.DrawLine} = 2 \\
\text{Form1.Line}(x_a, y_a)-(x_d, y_d), \text{vbBlue} \\
\text{Form1.Line}(x_b, y_b)-(x_e, y_e), \text{vbBlue} \\
\text{Form1.Line}(x_c, y_c)-(x_f, y_f), \text{vbBlue} \\
\text{Form1.DrawLine} = 0 \\
\text{Form1.DrawLine} = 5 \\
\text{If } beaf = 2 \text{ Then} \\
\text{Form1.DrawMode} = \text{vbCopyPen} \\
\text{Form1.PSet}(x_h, y_h), \text{QBColor}(pcl) \\
\text{Form1.DrawMode} = \text{vbNotXorPen} \\
\text{Else} \\
\text{Form1.PSet}(x_h, y_h), \text{QBColor}(pcl) \\
\text{End If} \\
\text{Form1.DrawLine} = 1 \\
\text{Else} \\
\text{If } bh < 0 \text{ Then} \\
dm &= x_a: x_a &= x_b: x_b &= dm \\
dm &= y_a: y_a &= y_b: y_b &= dm \\
\text{End If} \\
\text{If } ch < 0 \text{ Then} \\
dm &= x_a: x_a &= x_c: x_c &= dm \\
dm &= y_a: y_a &= y_c: y_c &= dm \\
\text{End If} \\
x_u &= x_b - x_a: u_y &= y_b - y_a \\
x_v &= x_a - x_c: v_y &= y_a - y_c \\
x_w &= x_c - x_b: w_y &= y_c - y_b \\
d &= w_x \times v_y - w_y \times v_x \\
\text{If } \text{Abs}(d) > 0.001 \text{ Then} \\
a &= \text{Sqr}(w_x \wedge 2 + w_y \wedge 2) \]
\begin{verbatim}
b = Sqr(vx ^ 2 + vy ^ 2)
c = Sqr(ux ^ 2 + uy ^ 2)
p1 = xb * yc - xc * yb; q1 = -wy * ya - wx * xa
q2 = -vy * yb - vx * xb
xd = (p1 * wy - q1 * wx) / a ^ 2
yd = (-p1 * wx - q1 * wy) / a ^ 2
xh = (q2 * wy - q1 * vy) / d
yh = (q1 * vx - q2 * wx) / d
H_Prolong xb, yb, xa, ya, ecl
H_Prolong xc, yc, xa, ya, ecl
Form1.DrawLine = 2
Form1.Line (xh, yh)-(xd, yd), vbBlue
Form1.Line (xb, yb)-(xh, yh), vbBlue
Form1.Line (xc, yc)-(xh, yh), vbBlue
Form1.DrawLineStyle = 0
Form1.DrawLineWidth = 5
If beaf = 2 Then
    Form1.DrawLineMode = vbCopyPen
    Form1.PSet (xh, yh), QBColor(pcl)
    Form1.DrawLineMode = vbNotXorPen
Else
    Form1.PSet (xh, yh), QBColor(pcl)
End If
Form1.DrawLineWidth = 1
End If
End If
End Sub

'---------------------------------------------------------------------
'
Extending of two sides adjacent at the vertex with an obtuse angle
'
'---------------------------------------------------------------------

Private Sub H_Prolong(x1, y1, x2, y2, ecl)
    Form1.DrawLineWidth = 1
    l = Sqr((x1 - x2) ^ 2 + (y1 - y2) ^ 2)
    ra(1) = Sqr((x1 - sx) ^ 2 + (y1 - sy) ^ 2)
    ra(2) = Sqr((x1 - ex) ^ 2 + (y1 - sy) ^ 2)
    ra(3) = Sqr((x1 - ex) ^ 2 + (y1 - ey) ^ 2)
    ra(4) = Sqr((x1 - sx) ^ 2 + (y1 - ey) ^ 2)

    r = ra(1)
    For i = 2 To 4
        If ra(i) > r Then r = ra(i)
    Next i

    p = r / l
    tx = (1 - p) * x1 + p * x2
    ty = (1 - p) * y1 + p * y2
    Form1.DrawLineStyle = 2
    Form1.Line (x2, y2)-(tx, ty), QBColor(ecl)
    Form1.DrawLineStyle = 0
End Sub
\end{verbatim}
Presenting a pedal triangle

Private Sub Pedal_triangle(xa, ya, xb, yb, xc, yc, col)
    ux = xb - xa: uy = yb - ya
    vx = xa - xc: vy = ya - yc
    wx = xc - xb: wy = yc - yb
    a = Sqr(wx ^ 2 + wy ^ 2)
    b = Sqr(vx ^ 2 + vy ^ 2)
    c = Sqr(ux ^ 2 + uy ^ 2)
    ah = b ^ 2 + c ^ 2 - a ^ 2
    bh = c ^ 2 + a ^ 2 - b ^ 2
    ch = a ^ 2 + b ^ 2 - c ^ 2
    p1 = xb * yc - xc * yb: q1 = -wy * ya - wx * xa
    p2 = xc * ya - xa * yc: q2 = -vy * yb - vx * xb
    p3 = xa * yb - xb * ya: q3 = -uy * yc - ux * xc
    xd = (p1 * wy - q1 * wx) / a ^ 2
    ye = (p2 * vy - q2 * vx) / b ^ 2
    xf = (p3 * uy - q3 * ux) / c ^ 2
    yf = (-p3 * ux - q3 * uy) / c ^ 2
    Form1.DrawWidth = 5
    Form1.PSet (xd, ye), QBColor(col)
    Form1.PSet (xe, ye), QBColor(col)
    Form1.PSet (xf, yf), QBColor(col)
    Form1.DrawWidth = 2
    Form1.Line (xd, ye)–(xe, ye), QBColor(col)
    Form1.Line (xe, ye)–(xf, yf), QBColor(col)
    Form1.Line (xf, yf)–(xd, ye), QBColor(col)
    Form1.DrawWidth = 1
    Orthocenter xd, yd, xe, ye, xf, yf, 9, 10, 2
End Sub